Labor market frictions and the Balassa-Samuelson model∗

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Abstract

This paper addresses the role of labor market frictions in the transmission process of sectoral productivities shocks to the relative price of nontradables. The Balassa-Samuelson model based on frictionless labor markets predicts (i) proportionality between relative prices and the cross-sectoral productivity differential and (ii) wage equalization across sectors. Using panel cointegration and unit root tests applied to a panel of fourteen OECD economies, our empirical evidence does not support these implications. This paper shows that these puzzles can be successfully explained by a two-sector model with labor market frictions. In particular, this paper considers two types of rigidities: labor reallocation costs across sectors and matching frictions similar to those found in the Mortensen-Pissarides model of unemployment.

Keywords: Balassa-Samuelson model, labor market frictions, productivity

JEL Classification: F31, F41, J64

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1 Introduction

The Balassa-Samuelson model (BS hereafter) has achieved workhorse status for the analysis of productivity shocks in the traded and nontraded goods sectors (Balassa [1964] and Samuelson [1964]). Its key insight is to show that, if labor market is frictionless, real wages are equalized across sectors and the relative price of nontradables is strictly proportional to the cross-sectoral productivity differential.\footnote{With competitive labor markets, wage rate equals the marginal product of labor, and with standard technologies, average and marginal products are proportional. Under these conditions, the slope of the production possibility curve, given by the relative price of nontradables, is equal to the ratio of the average products of labor in the two sectors.}

The voluminous literature exploring the internal workings of the BS model is a clear reflection of its success (see Froot and Rogoff [1995] for a survey). This is somewhat surprising given that the empirical validity of the standard model’s predictions are far from clear: the wage equalization assumption does not hold, both in the short and medium run, and, the relative price responsiveness to sectoral productivity shocks is generally found to be smaller than the unit-elasticity predicted by the theory.\footnote{For the wage equalization assumption, see Lee [1995], Strauss and Ferris [1996] and Strauss ([1997], [1998]). Skeptical evidence on the proportionality hypothesis is produced by De Gregorio et al. [1994] and Lee and Tang [2007], while Canzoneri et al. [1999] and Kakkar [2003] find some support to this hypothesis}

Our own empirical evidence reported in Section 2 confirms these findings and shows that the transmission of productivity shocks is likely to differ from that of the BS model. These apparent puzzles may reflect a restrictive way of modelling the labor market. In this paper, we argue that incorporating labor market frictions in the BS model can potentially account for the above evidence.

In the standard BS model, the labor market is perfectly competitive and reallocations of workers across sectors occur without frictions. In such environment, wages act as the main transmission mechanism and convert primitive shocks to technology into movements in the relative price. In particular, biased technological shocks favoring the traded sector bid up wages in that sector. Labor being perfectly mobile across sectors, wages in nontradables increase too. Because productivity gains in nontradables are smaller than those in tradables, non traded producers are only able to meet the higher wages by increasing their prices. While the assumption of Walrasian labor markets is convenient because of its simplicity, however, the BS model is unable to explain sectoral wage differentials and the tendency for relative productivities to grow by more than the relative price. In contrast, a two-sector general equilibrium model with labor market frictions has the potential to explain these findings in an otherwise standard setup.

In this paper, we extend the BS model with a more realistic labor market. In particular, we consider, separately, two alternative frictions in the form of (i) labor reallocation costs across sectors and (ii) matching frictions similar to those found in the Mortensen-Pissarides model of unemployment. The two types of labor market frictions considered here possess two particularly attractive attributes for the analysis of productivity shocks on prices and wages. First, the two formulations are quite explicit and have strong theoretical micro-foundations. As a result, the frameworks presented in this paper are highly tractable, allowing for a transparent analysis of productivity shocks and a clear identification of the underlying propagation mechanisms. Second, the two inefficiencies subsist both in the short and the long run, whereas nominal wage rigidities (Calvo-style wage setting or quadratic costs of adjusting nominal wages) only exists in the short run. Since most of the BS theoretical priors are based on long-run effects of productivity, these frictions provide well-suited extensions to illustrate the role of labor market rigidities.
Introducing labor market frictions, our key findings are as follows. First, wage differentials emerge endogenously, accommodating the evidence drawn up above. In the sluggish adjustment of labor model, these arise since the incentive to reallocate labor is dampened, because, in moving across sectors, workers incur costs. In the search economy, the link between marginal product and wages is broken, and wage differences are related to intersectoral differences in search costs. Second, the introduction of labor market frictions makes the relative wage vary negatively with productivity and breaks the unitary correlation between the relative price and the cross-sectoral productivity differential, results that are consistent with the data. In both specifications, wages in the traded sector are perfectly indexed to productivity gains in that sector while wages in the non traded sector increase less than one-for-one with productivity. As a result, the gap between sectoral earnings widens. When frictions take the form of reallocation costs, the limited upward pressure on wages in the non traded sector is related to the fact that the demand for labor in that sector falls by less as a consequence of the imperfect labor mobility. In the search model, higher wages in the non traded sector are associated with higher labor market tightness in this sector. Hence, employed workers receive only a share of the benefit from the increase of labor productivity (unemployed workers receive also a part), resulting in a lower wage increase compared to that observed in sector $T$. In both frameworks the relative wage responds negatively to technological shocks, an important phenomena which the BS model based on Walrasian labor markets is not designed to address. Moreover, as the relative wage absorbs part of the productivity growth, the upward pressure on the relative price of nontradables is limited. This implies that the relative price responsiveness to sectoral productivity shocks is smaller than the unit-elasticity predicted by the BS framework. Third, our models predict quantitative responses of relative wages and prices to asymmetric productivity disturbances that are in line with our empirical evidence, given plausible calibrations of the parameters governing frictions in the labor market. To assess our theoretical findings, we simulate our models and estimate the quantitative response of the relative price and the relative wage. Different values for labor market frictions parameters are employed as robust checks. Further, our numerical results show that the quantitative effects of productivity shocks on the relative price and the relative wage depend on the strength of labor market frictions such that the higher the rigidities the lower (higher resp.) the response of the relative price (relative wage resp.) to productivity shocks. Economies with frictionless labor markets may experience, therefore, higher price inflation (smaller wage inflation resp.).

A key aspect of our study is its focus on sectoral labor market frictions to explain the evolution of relative prices and wages. It is now well documented that rigidities in labor markets can differ across sectors due to intersectoral mobility costs or to differences in job creation/ destruction rates. First, Wolpin [2006] suggest that costs of reallocating labor between sectors may be substantial. Using survey data on US individuals, they find that the cost of moving from any occupation in the goods sector to the same occupation in the service sector ranges generally between 50 and 75% of average annual earnings. As an implication, between-sector employment shifts should be less pronounced than within-sector job flows when employees bear large mobility costs across sectors. Davis and Haltiwanger [1999] report some evidence for this finding and show that job movements most frequently take place within the same sector, not between different sectors. For example, using detailed industry classification (450 industries) they find that only 13% of job reallocation in the US is accounted by between-sector employment shifts. Second, recent empirical studies provide compelling evidence that search frictions vary across sectors. Davis and Haltiwanger [2006] find that quarterly average job creation and job
destruction flow rates vary widely among industries. For example, job flow rates are roughly three times larger in construction (14%) than in manufacturing (5%). Further, heterogeneity in unemployment rates across sectors is also an important stylized fact about the labor market in advanced economies. To illustrate, for the US in 2009, manufacturing, construction and education had an unemployment rate of 12.1%, 19% and 5.3% respectively (source: BLS).

Our focus on sectoral labor market frictions is related to a number of studies, both in international trade and international macroeconomics literature. Davidson et al. [1999] provide an analysis of international trade with labor markets that are characterized by two-sided search and matching frictions. In their model differences in labor market frictions, both across sectors and across countries, generate Ricardian-type comparative advantage.3 Wälde and Weiss [2006] extend the two-sector small open economy framework by incorporating matching frictions to study the effects of changes in the relative world market price. They show that these effects are unequally distributed across sectors: a drop of the relative of good implies a shrinking of that sector and an expansion of the other sector. Further, three interesting applications of intersectoral labor reallocation costs in the international macroeconomics literature are Agénor and Santaella [1998], Garcia-Cebro and Varela-Santamaria [2009] and Craighead [2009]. Agénor and Santaella [1998] and Garcia-Cebro and Varela-Santamaria [2009] examine the dynamics of a two-sector open economy to disinflation policies and monetary shocks, respectively, in the presence of labor adjustment costs, while Craighead [2009] shows that labor reallocation rigidities can substantially increase the real exchange rate volatility generated by two-country real business cycle model. Finally, most closely related to this work is Sheng and Xu [2009]. This paper uses a two-sector search-matching framework to investigate the effects of sectoral productivity improvement on real exchange rate. Our paper differs in several respects from Sheng and Xu [2009]. First, we expand the analysis of productivity shocks beyond the relative price of nontradables and focus on the effects on the relative wage as well. Second, our paper differs also in its quantitative emphasis. Our numerical simulations enrich our analytical analysis and quantify some workings of the models. Third, our analytical solutions are more explicit, which allow for a more suitable comparison of our predictions with that of the original BS framework. Finally, our econometric strategy, based on unit roots tests and cointegration, allows for a more formal evaluation of the predictions of the BS model. Sheng and Xu [2009] regress the change in the real exchange rate on changes in sectoral productivity differentials. However, the BS model is essentially a model that determines the real exchange rate in the long run. Hence it would be more appropriate to estimate the model in levels.

The remainder of the paper is structured as follows. Section 2 documents the long-run effects of relative productivity shocks on the relative wage and the relative price. Section 3 describes the theoretical two-sector models and provides some analytical insights into the transmission of relative productivity shocks shocks as well as the quantitative properties of the two theoretical specifications. The final Section 4 concludes.

2 The observed effects of sectoral technological shocks

The BS model has two important predictions for the behavior of relative wage (the wage equalization across sectors hypothesis) and relative price (the proportionality proposition between relative prices and the cross-sectoral productivity differential). We explore empirically these implications for a panel

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3This approach has been extended recently by Dutt et al. [2009] and Helpman and Itskhoki [2009].
of fourteen OECD countries from 1970 to 2007 in two ways. First, we test for the presence of unit roots in the relative wage and in the difference between the relative price and the relative labor productivity ratio. If the BS principles hold in the long-run, these variables should be stationary. Next, we estimate the long-run effects of relative productivity shocks on the relative wage and relative price by running the following cointegrating regressions:

\[
\begin{align*}
\ln\left(\frac{w_{i,t}}{w_{T,t}}\right) &= \delta_{0,i} + \beta_w \ln\left(\frac{A_{T,i,t}}{A_{N,i,t}}\right) + v_{i,t}, \\
\ln\left(\frac{p_{i,t}}{p_{T,t}}\right) &= \alpha_{0,i} + \beta_p \ln\left(\frac{A_{T,i,t}}{A_{N,i,t}}\right) + u_{i,t},
\end{align*}
\]

where \( i \) and \( t \) index country and time, \( \delta_{0,i} \) and \( \alpha_{0,i} \) are the deterministic components and \( v_{i,t} \) and \( u_{i,t} \) the i.i.d. error terms. \( w_{i,t}, p_{i,t} \) and \( A_{i,t} \) denote wage rate, price and labor productivity per worker respectively (traded sector variables are labelled \( T \) and non traded \( N \)). Under the textbook version of the BS model, the slope coefficient in wage regression \((1a)\) should be zero \((\hat{\beta}_w = 0)\), while the slope in price equation \((1b)\) should be equal to unity \((\hat{\beta}_p = 1)\).

We start by examining the stochastic properties of the variables \( p_{i,t} \) (\(\equiv p_{i,t}^N/p_{i,t}^T\)), \( w_{i,t} \) (\(\equiv w_{i,t}^N/w_{i,t}^T\)) and \( A_{T,i,t}/A_{N,i,t} \). In order to test for the presence of unit root, we carry out the panel tests proposed by Levin et al. [2002], Breitung [2000], Im et al. [2003], Fisher-type tests using ADF and PP tests (Maddala and Wu [1999]) and Hadri [2000], with results displayed in Table 1.6

<table>
<thead>
<tr>
<th>Variable</th>
<th>LLC (t-stat)</th>
<th>Breitung (t-stat)</th>
<th>IPS (W-stat)</th>
<th>MW (ADF)</th>
<th>MW (PP)</th>
<th>Hadri (Z(\mu)-stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{i,t} )</td>
<td>0.68</td>
<td>0.76</td>
<td>1.00</td>
<td>1.00</td>
<td>0.91</td>
<td>0.00</td>
</tr>
<tr>
<td>( w_{i,t} )</td>
<td>0.01</td>
<td>0.65</td>
<td>0.33</td>
<td>0.12</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>( A_{T,i,t}/A_{N,i,t} )</td>
<td>1.00</td>
<td>0.41</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( p_{i,t} - A_{T,i,t}/A_{N,i,t} )</td>
<td>0.53</td>
<td>0.22</td>
<td>0.99</td>
<td>0.95</td>
<td>0.98</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: LLC and Breitung are the t-statistic developed by Levin et al. [2002] and Breitung [2000] respectively. IPS corresponds to the Im, Pesaran and Shin’s [2003] \(W_{bar}\) test. MW (ADF) (MW (PP) resp.) denote the Maddala and Wu’s [1999] \(P\) test based on Augmented Dickey-Fuller (Phillips-Perron resp.) \(p\)-values. Hadri corresponds to the Hadri’s [2000] \(Z_{\mu}\) test.

All unit root tests applied to the relative price \((p_{i,t})\) and the labor productivity ratio \((A_{T,i,t}/A_{N,i,t})\) show that non stationarity is pervasive. Further, on the basis of all tests, except for the Levin et al. [2002] unit root test, the relative wage variable \((w_{i,t})\) is found to be nonstationary too. Thus, the data strongly reject the assumption of wage equalization across sectors postulated by the BS framework due to the assumption of frictionless labor markets. If the theoretical assumptions underlying this model were correct, the relative wage should be stationary as any sectoral wage differential should be quickly eroded thanks to the perfect mobility of labor within the economy. On the contrary, as the null hypothesis of a unit root can not be rejected for \(w_{i,t}\), the empirical evidence reported in Table 1 makes clear that sectoral wages differentials persist for substantial period of time. The next question

4The data set and construction of variables are described in more details in Appendix A.

5It is worth emphasizing that the productivity proxy variable \(A_{i,t}\) is labor productivity rather than total factor productivity (TFP). Canzoneri et al. [1999] argue that the use TFP is subject to two severe limitations: (i) it involves data on sectoral capital stock and estimates of labor’s share in production that are less reliable than data on sectoral labor and output, and (ii) it is generally associated with a Cobb-Douglas production function while the proportionality proposition holds for a broader class of technologies that are much less restrictive than the Cobb-Douglas function.

6All tests, except for the Hadri test, are based on the null of a unit root against the alternative of trend stationarity.
to be addressed is whether relative price and cross-sectoral productivity differential are proportional in the long-run. This is done by constructing the difference \( (p_{i,t} - (A_{i,t}^T/A_{i,t}^N)) \) and applying the same unit root tests to the variable so obtained. The evidence reported in the last row of Table 1 indicates that the proportionality hypothesis is unanimously rejected, suggesting that the relative price does not reflect entirely the relative productivity ratio.

As a second step, we test whether \( p_{i,t} \) and \( w_{i,t} \) are cointegrated with \( A_{i,t}^T/A_{i,t}^N \) and whether the cointegrating slope is zero in equation (1a) and one in (1b). For the two combinations of variables in equation (1), the null hypothesis of no cointegration is mostly rejected according to the different tests suggested by Pedroni ([1999], [2004]). Cointegrating relationships are based on the group-mean fully modified OLS (FMOLS) and dynamic OLS (DOLS) procedures for cointegrated panel proposed by Pedroni ([2000], [2001]), with results reported in Table 2.

Table 2: Prices, wages and productivity differentials

<table>
<thead>
<tr>
<th>Eq.</th>
<th>DOLS</th>
<th>FMOLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta}_w ) (1a)</td>
<td>-0.240*</td>
<td>-0.236*</td>
</tr>
<tr>
<td>( \hat{\beta}_p ) (1b)</td>
<td>0.685*</td>
<td>0.680*</td>
</tr>
<tr>
<td>( t(\hat{\beta}_p = 1) )</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: * denotes significance at 1% level. The last row reports the p-value of the test of \( H_0: \hat{\beta}_p = 1 \).

Consistent with previous results, the coefficient estimate for the wage equation (1a) exhibits a pattern that is not supportive of the wage equalization hypothesis. Contrary to the BS model for which the regression of \( w_{i,t} \) on \( A_{i,t}^T/A_{i,t}^N \) should lead to an insignificant coefficient, productivity differentials have a statistically significant negative effect of the relative wage (with a point estimate about -0.24). The results of the price regression (1b) invalidate the other building block of the BS model. The coefficient on relative productivity in tradables and nontradables has the predicted sign and is highly significant (with a coefficient estimate of 0.68). Nonetheless, it is inappropriate to conclude that the BS model is successful just because the slope in the regression (1b) is positive. Indeed, the results displayed in the last row of Table 2, indicate that the restriction that slope of the cointegrating vector \( \hat{\beta}_p \) is equal to the unit implied by the theory is strongly rejected using a 1% significance level. Exogenous shocks to relative technology are, therefore, not fully transmitted to the relative price.

To sum up, in the BS paradigm with perfectly competitive labor market, the relative price of nontradables is proportional to relative labor productivity and the relative wage is constant. Our analysis makes clear that the empirical evidence can hardly be interpreted as favorable to these implications. By contrast, we view the results reported in Tables 1 and 2 as suggesting that labor market rigidities are crucial to understand the observed patterns of relative prices and relative wages. The virtue of

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7Pedroni ([1999], [2004]) considers seven tests based on the estimated residuals. Four (three resp.) come from pooling data along the within (between resp.) dimension. Results are summarized in Table 5 in Appendix A.

8The DOLS estimator adds \( q \) leads and lags of \( \Delta (A_{i,t}^T/A_{i,t}^N) \) as additional regressors in (1). The parameter \( q \) was set to one, however, our results were identical when \( q = 2 \) or \( q = 3 \). As robustness checks, we also used alternative estimators: dynamic fixed effects estimator, mean group estimator (Pesaran and Smith [1995]) and pooled mean group estimator (Pesaran et al. [1999]). The results were very similar too and are available upon request from the authors.

9The literature is rich in alternative explanations to the aforementioned puzzles: capital market imperfections (Rogoff [1992]), endogenous tradability (Bergin et al. [2006]), entry/exit of firms (Ghironi and Melitz [2005]), spatial distribution of firms (Méjean [2008]). Our paper explores an alternative approach based on labor market frictions.
those rigidities, like search frictions or sluggish reallocation of workers, is that it generates differences in wages across sectors and, thereby, does not constraint the relative price of nontradables to be strictly proportional to the cross-sectoral productivity differential. In the next Section we address these issues in the context of fully specified two-sector dynamic general equilibrium models.

3 Two-sector models with labor market frictions

Consider an open economy that produces and consumes a tradable good and a nontradable good. Traded sector aggregates are labelled $T$ and non traded $N$. Further, denote the price of the two goods as $p^T$ and $p^N$ respectively. The price of the traded good is determined on the world market and is exogenously given to the domestic economy. The latter price is used as the numeraire. In this setting, $p$ stands for the relative price of nontradables ($p^T = p^N/p^T$). On the demand side, the agent consumes both tradable, $c^T$, and nontradable goods, $c^N$. In addition, he holds a stock of foreign assets $b$ (denoted in terms of the traded good) which bears a real rate of return given by the exogenous world interest rate $r^*$. The production side consists of representative firms, each of which uses only labor as an input. In sector $j = T, N$, each firm employs $L^j$ workers and pays them the wage $w^j$. The output $Y^j$ is obtained from the technology $Y^j = A^jL^j$, where $A^j$ is the labor productivity in sector $j$.

The two alternative model specifications presented below build on the above assumptions by incorporating labor market frictions modelled as intersectonal labor reallocation costs and matching frictions. The first specification features intersectonal labor adjustments costs that prevent instant reallocation of the labor force. The second extension embeds the BS framework in a standard search model of unemployment. The crucial element in these extensions is an endogenous sectoral wages differential, accounting, therefore, for one of the stylized facts highlighted previously. To keep the models as close as possible to the standard BS setting, we abstract from nominal rigidities and imperfections in the goods and financial markets (even though we recognize their importance). Thus, apart from the labor market, all other markets operate as Walrasian auctions.

3.1 The model with labor reallocation costs

The representative household gains utility from consumption and experiences disutility from supplying effective labor effort. He seeks to maximize the objective function

$$\int_0^\infty \left( c^{1 - 1/\sigma} - \frac{1}{1 - 1/\sigma} - \frac{1}{1 + 1/\sigma} \right) e^{-\beta t} dt,$$

where $\beta \in [0, 1]$ denotes the consumer’s discount rate, $\sigma > 0$ the intertemporal elasticity of substitution, $\sigma_L > 0$ the Frisch elasticity of labor supply and $\gamma > 0$ a scaling parameter of disutility of work.\(^{10}\) The arguments $c$ and $L$ denote the household’s consumption and quantity of labor supplied respectively.\(^11\)

The composite consumption good $c$ is a CES aggregate of traded and non traded consumptions:

$$c = \left( (c^T)^{1 - 1/\phi} + (c^N)^{1 - 1/\phi} \right)^{\phi/(\phi - 1)}.$$

\(^{10}\)Note that all variables are functions of time $t$. The time-argument is omitted except when it is required by the context. In addition, time derivatives are denoted by a dot above the variable concerned ($\dot{x} \equiv dx/dt$).

\(^{11}\)The felicity function (2) is additively separable in consumption and labor. This specification is not consequential for our results we obtain since our theoretical predictions are robust to more general non-separable utility functions.
where \( \phi > 0 \) denotes the elasticity of substitution between tradables and nontradables. In order to cover the empirically most relevant case, we assume that tradables and nontradables are imperfect substitutes in consumption \( (\phi > 1) \). Ostry and Reinhart [1992] provide estimates of \( \phi \) for developing economies in the range of 1.22 to 1.28. For the major industrialized countries, Cashin and McDermott [2003] find that the elasticity of substitution between non traded goods and traded goods ranges from 0.63 to 3.50, averaging 2.2. These estimates are somewhat higher than those obtained by Stockman and Tesar [1995] and Mendoza [1995] (0.44 and 0.74 respectively). A possible explanation to this difference relates to the restrictive assumption employed in the latter studies. Specifically, they estimate \( \phi \) by running the regression \( \ln(\frac{c_N}{c_t}) = \alpha_0 + \phi \ln(p_t) + \alpha_1 \ln(y_t) + \epsilon_t \), where \( \frac{c_N}{c_t} \) is the non traded good expenditure share, \( p_t \) the price index for nontradables, \( y_t \) per capita GDP (to pick up income effects) and \( \epsilon_t \) is a random disturbance. The coefficient on the relative price, \( \phi \), is the intratemporal elasticity of substitution. Because this estimate is not structural, i.e. not policy-invariant, it would necessarily change whenever policy or macroeconomic context change, especially when the profile of relatives prices is not constant through time. By contrast, Ostry and Reinhart [1992] and Cashin and McDermott [2003] have attempted to incorporate this criticism and estimate the structural parameters of interest from the Euler equations consistent with an intertemporal optimizing two-good model of consumption.

The second term in the period utility function (2) is disutility from total hours worked \( L \), which is a function of labor supply to each sector \( L^j \). Following Casas [1984] and Horvath [2000], the aggregate labor index takes the form:

\[
L = \left( \left( \frac{L^T}{1+1/\epsilon} \right)^{1+1/\epsilon} + \left( \frac{L^N}{1+1/\epsilon} \right)^{1+1/\epsilon} \right)^{\epsilon/(\epsilon+1)},
\]

where the parameter \( \epsilon > 0 \) reflects the ease with which the quantities of labor supplied to tradables \( L^T \) and to nontradables \( L^N \) can be substituted for each other. The representative worker cares not only about the overall amount of hours he supplies but also about the sectoral distribution of his labor effort. The worker has therefore a preference for diversity which represents the utility gain that is obtained from spreading a certain amount of labor over the two sectors rather than concentrating it on a single sector, even in the presence of wage differences across sectors. More formally, the parameter \( \epsilon \) represents the cost of reallocating labor between sectors. For \( \epsilon < \infty \), hours worked in tradables and nontradables are not perfect substitutes from the perspective of the representative worker. An interpretation of this is that the cost of moving of unit of labor between sectors is positive, such that the representative agent is unwilling to reallocate his effort across sectors. The intersectorally mobility in the labor market is, thereby, moderated. In the limiting case (as \( \epsilon \to \infty \)), labor hours in each sector turn out to be perfect substitutes and thereby are identical goods for the worker. Hence, labor is perfectly mobile and all sectors pay the same wage.

The motivation for the specification (4) is twofold. First, this formulation introduces costs of reallocating labor between sectors without deviating from the representative agent assumption and, also, without imposing quadratic adjustment costs on intersectoral labor movements. Even though the use of the quadratic function to specify adjustment costs has attractive implications, it suffers from the serious disadvantage that it leads to unnecessary mathematical complications.\(^{13}\) This drawback motivates our alternative approach in which the costs for reallocating labor across sectors take the form of utility loss when the amount of labor supplied to one sector is changed. In doing so, the

\(^{12}\)The implications of relaxing it will be discussed below.
\(^{13}\)The quadratic structure of intersectoral adjustment costs is studied in Georges [1995] and Craighead [2009] for labor and in Musa [1978] and Morshed and Turnovsky [2004] for capital.
model gains tractability. Second, the formulation (4) considers explicitly the full range of degrees of intersectoral labor mobility. Indeed, by appropriately parameterizing the degree of costliness $\epsilon$, the situations of perfect immobility ($\epsilon = 0$) and perfect mobility ($\epsilon \to \infty$) of labor emerge as special cases.

The household’s income is composed of wages received in each of the two sectors, $w^j L^j$, and interest earnings from holding traded bonds $r^* b$. The representative agent uses his income for purchasing both the traded good and the non traded good, and accumulating bonds on financial markets. He chooses sequences $\{c^T, c^N, L^T, L^N, b\}$ to maximize the objective function (2) subject to the flow budget constraint (expressed in terms of the traded good):

$$\dot{b} = r^* b + w^T L^T + w^N L^N - c^T - pc^N.$$  \hspace{1cm} (5)

Letting $\lambda$ be the marginal utility of wealth, the household’s first-order conditions can be condensed to

$$\dot{\lambda} = \lambda (\beta - r^*),$$  \hspace{1cm} (6a)

$$\frac{c^T}{c^N} = p^\phi;$$  \hspace{1cm} (6b)

$$\frac{L^N}{L^T} = \left(\frac{w^N}{w^T}\right)^\epsilon;$$  \hspace{1cm} (6c)

and the transversality condition $\lim_{t \to \infty} \lambda b(t)e^{-\beta t} = 0$. The dynamic part of the solution is contained in (6a) which is the Euler equation. For an interior solution, we require the time preference rate to be equal to the exogenously given interest rate. This standard assumption made in the literature implies that the marginal utility of wealth must remain constant over time, that is, $\lambda = \bar{\lambda}$. Condition (6b) determines the intratemporal allocation of consumption expenditures between tradables and nontradables and shows that relative demand depends only on the relative price. The third efficiency condition (6c) is a crucial feature of the introduction of imperfect labor mobility. This equation asserts that, in equilibrium, the marginal rate of substitution between an additional unit of labor in the non traded sector and in the traded sector is positively related to the relative wage $w(\equiv w^N/w^T)$. The presence of labor adjustments costs, represented by the parameter $\epsilon$ is the source of the sectoral wage differential. Indeed, if costs associated with switching labor across sectors are high, workers will not find optimal to reallocate their labor supply unless the wage differential across sectors is large enough to compensate the disutility generated by these costs. Thus, intermediate values of $\epsilon$ imply that labor mobility is imperfect. The associated consequence is that the wage differential is persistent between the two sectors as the sluggish adjustment of labor does not allow to correct sectoral earnings discrepancies. By contrast, if labor reallocation is costless (i.e. $\epsilon \to \infty$), equation (6c) gives the immediate result that wages are equalized between sectors ($w^T = w^N$ such that $w^c = 1$).

The production side is simple. Profit maximization in sector $j = T, N$ by perfectly competitive firms leads to the standard pricing rule that equalizes labor marginal product to wage rate, $p^j A^j = w^j$. Finally, the resource constraint for nontradables requires that output equals consumption, $Y^N = c^N$. Combining this condition with the flow budget constraint of the representative agent (5), we obtain the market clearing condition for tradables, i.e. the current account dynamics:

$$\dot{b} = r^* b + Y^T - c^T.$$  \hspace{1cm} (7)

---

14See Turnovsky [1997] and references therein. The agent’s budget constraint (5) can be combined with first-order conditions to solve for the equilibrium value of the marginal utility of wealth: $\bar{\lambda} = \left[ (Y^T + pY^N)/(\pi^c)^{1-\sigma} \right]^{-1/\sigma}$ where $\pi^c$ denotes the price of the consumption bundle $c$ in terms of the traded good ($\pi^c \equiv (1 + p^{1-\phi})^{1/(1-\phi)})$, the details of derivation are reported in an Appendix which is available on request.
This dynamic equation can be linearized to determine the general solution for the stock of foreign assets. Employing standard techniques and invoking the transversality condition, the stable path for \( b(t) \) consistent with long-run solvency of the economy is \( b(t) = b_0 \), where \( b_0 \) is the initial stock of foreign assets. For analytical convenience, the economy has no outstanding net foreign assets at the initial equilibrium (i.e. \( b_0 = 0 \)). Hence, the resource constraint for tradables reads \( Y^T = c^T \).

An equilibrium for this economy consists of an allocation \( \{ c^T, c^N, p, w, L^T, L^N \} \) that satisfies: (i) the household’s first-order conditions (6b)-(6c), (ii) the firms’ optimal condition \( p^j A^j = w^j \) for \( j = T, N \) and (iii) the resource constraints for nontradable and tradable goods \( Y^j = c^j \) for \( j = T, N \). This equilibrium features three exogenous parameters: the relative productivity ratio \( A^T/A^N \), the elasticity of substitution between tradables and nontradables, \( \phi \), and the degree of labor market rigidity \( \epsilon \).

### 3.1.1 The Balassa-Samuelson effect with imperfect labor reallocation

In this Section, the mechanism by which the introduction of labor reallocation costs may alter the transmission process of productivity shocks is analyzed in detail. To characterize the response of the relative price, \( p \), and the relative wage, \( w \), the above equilibrium conditions are log-linearized around the steady-state, to give

\[
\hat{p} = \left( \frac{\epsilon + 1}{\epsilon + \phi} \right) (\hat{A}^T - \hat{A}^N), \tag{8a}
\]

\[
\hat{w} = \left( \frac{1 - \phi}{\epsilon + \phi} \right) (\hat{A}^T - \hat{A}^N), \tag{8b}
\]

where hats denote percentage deviations from initial steady-state. Let \( \zeta_p \equiv (\epsilon + 1)/(\epsilon + \phi) \in [0, 1] \) (\( \zeta_w \equiv (1 - \phi)/(\epsilon + \phi) \leq 0 \) resp.) denote the elasticity of the relative price (relative wage resp.) with respect to the ratio of relative productivity \( A^T/A^N \). Recall that our model nests the standard BS framework, which can be obtained by imposing the restriction \( \epsilon \to \infty \). In this case, the elasticity \( \zeta_p \) tends to one, so that technological shocks are fully transmitted to the relative price. Moreover, the right-hand side of equation (8b) collapses to zero so that \( \zeta_w = 0 \). In words, the relative wage is totally invariant to productivity shocks. This invariance is the result of the wage equalization hypothesis and crucially relies on the assumption of homogenous labor markets.

Now, consider the two-sector specification with imperfect labor reallocation. The introduction of costs of moving labor across sectors makes the relative wage vary negatively with productivity according to (8b) since \( \zeta_w \) is unambiguously negative. The amplitude of this elasticity is decreasing in the degree of labor mobility \( \epsilon \). That is, labor mobility costs amplifies (in absolute term) the effect of productivity increase on the relative wage. The intuition behind this result is as follows. By the law of one price, a biased technological shock favoring the traded sector (\( \hat{A}^T > \hat{A}^N \)) has a positive effect on wages in that sector (\( \hat{w}^T = \hat{A}^T \)). In the absence of reallocation costs, workers would instantly move from the nontradable sector to the tradable sector and bid wages upward in the non traded sector, until sectoral earnings were equalized (\( \hat{w}^N = \hat{w}^T = \hat{A}^T \)). With adjustment costs in the labor market instead, the incentive to reallocate labor towards the traded sector is reduced since in migrating across sectors, workers incur costs. It results that supply for labor in the non traded goods sector falls by less and the upward pressure on wages in that sector is damped. The traded goods sector experiences, therefore, the higher wage inflation, and the wage growth differential (\( \dot{w} = \hat{w}^N - \hat{w}^T \)) becomes negative such that \( \zeta_w < 0 \).
As the relative wage absorbs part of the productivity growth, the upward pressure on the relative price of nontradables is limited. This implies that the elasticity of the relative price to technological shocks is smaller in the model with labor reallocation costs than the unit elasticity predicted by the BS framework ($\zeta_p < 1$), a result that is consistent with the data. Moreover, the extent to which biased technological shocks towards the traded sector pushes the relative price upward is increasing in the degree of labor mobility $\epsilon$. When substantial adjustment costs impede the sectoral reallocation of labor ($\epsilon$ is low), the relative price response to changes in sector-specific technological shocks is smaller.

In summary, these results suggest that labor market frictions modelled as sectoral reallocation costs could help the BS model in replicating the response of relative price and relative wage to technological shocks predicted by the empirical evidence reported in Section 2. The following proposition states the aforementioned results formally.

**Proposition 1** Consider the two-sector model with labor reallocation costs. The elasticity of the relative price of nontradables and the relative wage with respect to the ratio of relative productivity are denoted by $\zeta_p$ and $\zeta_w$ and are given by (8a) and (8b) respectively. For any $\epsilon > 0$ and $\phi > 1$:

(i) $0 < \zeta_p < 1$,

(ii) $\zeta_w < 0$.

**Proof** See equations (8a)-(8b). ■

From the foregoing discussion, the impact of technological shocks hinges on the degree of labor mobility across sectors. For this to be the case, the substitutability in consumption of tradables and nontradables with $\phi > 1$ is a necessary condition. More precisely, for a given degree of labor market rigidity $\epsilon$, a low value of $\phi$ magnifies the rise in the relative price of nontradables and softens the drop in the relative wage in response to technology shocks. The explanation relies upon the willingness of households to engage in intratemporal substitution when facing sectoral technological shocks. For low values of $\phi$ households are reluctant to change the distribution of their consumption basket between tradables and nontradables in response to exogenous shocks to productivity. Given that behavior, clearing the markets for both goods requires that the relative supply of goods, and thereby, the relative demand for labor across sectors, respond weakly to the exogenous shock. Consequently, the labor market equilibrium is strongly immunized from productivity changes for low values of $\phi$, such that the relative wage response is limited. This in turn magnifies the effects of productivity gains on the relative price of nontradables for the firms' first-order condition to be fulfilled.

To illustrate the quantitative role of sectoral reallocation costs in propagating sectoral productivity shocks, we implement the model numerically. For this purpose, the general equilibrium based on (8a) and (8b) is computed by using parameter values for $\phi$ and $\epsilon$ taken from the literature. Reference estimates of the intratemporal elasticity of substitution between tradables and nontradables $\phi$ are provided by Cashin and McDermott [2003]. There is a considerable heterogeneity in their estimates, which vary from a low 0.63 for New Zealand to a high of 3.50 for the US, with a mean value across countries of 2.2. Consequently, we provide some sensitivity analysis of our results using values for $\phi$.

\[\text{If } \phi = 1, \text{ the model model collapses to the standard BS framework (}\zeta_p = 1 \text{ and } \zeta_w = 0).\]

\[\text{In that situation, quantitative effects of productivity shocks are independent of the degree of mobility of workers } \epsilon.\]

\[\text{More precisely, Cashin and McDermott [2003] report estimates about 1.96 for Australia, 3.40 for Canada, 0.63 for New Zealand, 1.70 for the UK and 3.50 for the US.}\]
between 1.5 and 3.5. We characterize the degree of labor mobility in terms of the elasticity $\epsilon$, for which the empirical evidence is quite scant. To the best of our knowledge, only Horvath [2000] provides some estimates. Using data on US sectoral employment, he reports $\epsilon = 1$. On this basis, we perform a sensitivity analysis with respect to $\epsilon$ in the range 0.5 to 3.

Table 3: Responses of $\tilde{p}$ and $\tilde{w}$ to changes in $A_T/A_N$ (labor reallocation costs model)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\epsilon$ = 0.5</th>
<th>$\epsilon$ = 1</th>
<th>$\epsilon$ = 2</th>
<th>$\epsilon$ = 3</th>
<th>BS</th>
<th>$\epsilon$ = 0.5</th>
<th>$\epsilon$ = 1</th>
<th>$\epsilon$ = 2</th>
<th>$\epsilon$ = 3</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.75</td>
<td>0.80</td>
<td>0.86</td>
<td>0.89</td>
<td>1</td>
<td>-0.25</td>
<td>-0.20</td>
<td>-0.14</td>
<td>-0.11</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.60</td>
<td>0.67</td>
<td>0.75</td>
<td>0.80</td>
<td>1</td>
<td>-0.40</td>
<td>-0.33</td>
<td>-0.25</td>
<td>-0.20</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>0.50</td>
<td>0.57</td>
<td>0.67</td>
<td>0.73</td>
<td>1</td>
<td>-0.50</td>
<td>-0.43</td>
<td>-0.33</td>
<td>-0.27</td>
<td>0</td>
</tr>
<tr>
<td>3.0</td>
<td>0.43</td>
<td>0.50</td>
<td>0.60</td>
<td>0.67</td>
<td>1</td>
<td>-0.57</td>
<td>-0.50</td>
<td>-0.40</td>
<td>-0.33</td>
<td>0</td>
</tr>
<tr>
<td>3.5</td>
<td>0.38</td>
<td>0.44</td>
<td>0.55</td>
<td>0.62</td>
<td>1</td>
<td>-0.63</td>
<td>-0.56</td>
<td>-0.45</td>
<td>-0.38</td>
<td>0</td>
</tr>
</tbody>
</table>

| Empirical estimates | $\hat{\beta}_p = 0.68$ | $\hat{\beta}_w = -0.24$ |

Notes: BS: Balassa-Samuelson model ($\epsilon \to \infty$).

Table 3 summarizes the numerical values for both the relative price ($\zeta_p$) and the relative wage ($\zeta_w$) responses. As the results shown in the Table make clear, the BS model suffers from one weakness: it overestimates the response of both the relative price and the relative wage. The introduction of imperfect labor mobility improves the model’s behavior along this dimension. Because of the sluggish adjustment of labor across sectors, the model captures, now, the fact that, for reasonable values of the parameters, the elasticity of the relative wage to sectoral productivity shocks is negative and falls in the range [-0.20; -0.40], close to empirical estimates. It also matches the response of the relative price to technological shocks. One additional finding seems worth noting. For the most empirical relevant calibration ($\epsilon = 1$ and $\phi = 2.2$), the match between the data estimated price elasticity to technological shock ($\hat{\beta}_p = 0.68$) and the model-generated elasticity ($\zeta_p = 0.63$) is quite striking, given the relatively basic structure of the framework.

### 3.2 The model with search frictions

The second extension considered here embeds the Pissarides [2000] search-theoretic framework in a two-sector general equilibrium model, which features search cost asymmetries across sectors. The search model we employ is standard in most of its components. We deviate from the standard search and matching model in that we assume that hiring of workers is proportional to the worker’s productivity in each sector rather than being constant as in most models. This formulation follows Hagedorn and Manovskii [2008] and aims at capturing the notion that it is more costly to hire more productive workers. Another virtue of this specification is that costs of a typical firm rise along with productivity gains and ensures thereby the existence of a meaningful steady-state (see Pissarides [2000]). Formally, a typical firm in sector $j$ posts vacancies to attract workers at a cost $\delta^j$ (in terms of the numeraire good). This vacancy posting cost is assumed to be linear in worker’s productivity, $\delta^j = \gamma^j A^j$, where $\gamma^j$ is a positive constant.

It is also assumed that the labor market frictions vary across sectors due to the fact that it may be more difficult to match workers with firms in one sector than in other. This makes the labor market segmented across sectors. While employed workers might be attached to the sector in which they work
due to frictions, searching unemployed individuals are assumed to be perfectly mobile across sectors. They have no particular attachment to a specific sector and randomly take the first job available. However, at any point of time, an unemployed worker can only search in one particular sector and, its search decision identifies the sector to which it belongs.

### 3.2.1 The labor market equilibrium

Matches in sector $j = T, N$, between job-seeking workers and vacancy-offering firms are formed according to a constant-returns to scale matching function

$$ M^j(V^j, U^j) = m^j(V^j)^{\xi}(U^j)^{1-\xi}, \quad (9) $$

where $U^j$ represents the number of searching unemployed individuals, $V^j$ is the total number of posted vacancies, $m^j > 0$ is match efficiency and $\xi \in ]0, 1]$ is a parameter capturing the vacancy intensity of the matching function (assumed to be common to both sectors). The matching function is homogenous of degree one, strictly increasing and concave in both arguments. Homogeneity implies that the vacancy matching rate is $q(\theta^j) \equiv M^j/V^j = m^j(\theta^j)^{\xi-1}$ which is decreasing in the degree of labor market tightness $\theta^j \equiv V^j/U^j$ (a greater ratio means a tighter labor market). Thus, the job-finding rate is given by $M^j/U^j = m^j(\theta^j)^{\xi} \equiv \theta^j q(\theta^j)$.

Labor market tightness $\theta^j$, the matching technology (9), together with the exogenous rate of job destruction in sector $j$, $\lambda^j$, condition the dynamics of the sectoral unemployment rate $u^j$:

$$ \dot{u}^j = \lambda^j (1-u^j) - \theta^j q(\theta^j) u^j. \quad (10) $$

Changes in unemployment result from a difference between the flow of workers whose lose their job, $\lambda^j (1-u^j)$, and the flow of job-seeking workers who find a job, $\theta^j q(\theta^j) u^j$. In steady-state, flows in employment are equal to flows out, and the stationary value of the unemployment rate is given by:

$$ \tilde{u}^j = \frac{\lambda^j}{\lambda^j + \theta^j q(\theta^j)}, \quad (11) $$

where steady-state values are denoted by means of a tilde overstrike.

The crucial decision of firms concerns the supply of jobs on the labor market. For convenience, each firm has at most one job that can be either vacant or filled. When it is filled, the representative firm in sector $j$ undertakes production using the constant-returns to scale production function $Y^j = A^j L^j$ and. As before, labor is the sole factor of production but hiring is now subject to recruitment costs $\delta^j$. For the sake of simplicity, it is useful to solve the firms’ optimization problem in asset value terms. Let $J^j_V$ define present discounted value of a vacant job and $J^j_J$ is the asset value of a occupied job in sector $j = T, N$. Let $r^*$ be the discount rate, the Bellman equation governing $J^j_V$ and $J^j_J$ satisfy:

$$ r^* J^j_V = -\delta^j + q(\theta^j)(J^j_J - J^j_V), \quad (12a) $$

$$ r^* J^j_J = p^j A^j - w^j + \lambda^j (J^j_V - J^j_J), \quad (12b) $$

where the bargained wage $w^j$ is taken as given by firms. The relation (12a) equates the capital cost of a vacant job $r^* J^j_V$ to its return on the labor market. The flow return comprises negative search costs ($-\delta^j$) and the expected capital gain associated with the change from the vacant to the filled

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17 The destruction rate $\lambda^j$ has not to be confused with the marginal utility of wealth $\lambda$, which is not superscripted.
state \( (J^*_j - J^j_U) \) which occurs with probability \( q(\theta^j) \). Similarly, equation \( 12b \) indicates that the flow capital cost of a job \( r^*J^j_J \) is equal to the return from the filled job, which consists of two parts: the profit created in production \( (p^j A^j - w^j) \) and the expected capital loss due to job destruction \( (J^1_V - J^j_j) \) with a probability \( \lambda^j \). Free entry in job creation drives the present value of a vacancy to zero, such that \( J^1_V = 0 \). Inserting this in \( 12a \) implies the following

\[
J^j_j = \frac{\delta^j}{q(\theta^j)}. \tag{13}
\]

The implicit assumption here is that firms decide to create jobs whenever the value of a vacancy is positive and thus potential profits will be eroded quickly by free entry. Making use of equation \( 13 \) to substitute \( J^j_j \) out of \( 12b \), the following equation can be derived

\[
p^j A^j - w^j = \frac{(r^* + \lambda^j) \delta^j}{q(\theta^j)}. \tag{14}
\]

The right-hand side of \( 14 \) represents the surplus created in production \( (p^j A^j - w^j) \), this is the profit the firm earns after labor has been paid. The left-hand side is the expected capitalized value of the firm’s hiring cost. Relation \( 14 \) conveys that these quantities must be equal at the optimum and implies, therefore, a wedge between marginal productivity of labor and the wage rate. If there were no hiring costs \( (\delta^j = 0) \), equation \( 14 \) would yield the standard condition for labor \( p^j A^j = w^j \).

By repeating the flow value approach used to solve the firms’ optimization problem, the behavior of workers can be summarized by the following Bellman equations:

\[
\begin{align*}
  r^*W^j_E & = w^j + \lambda^j(W^j_U - W^j_E), \tag{15a} \\
  r^*W^j_U & = \theta^j q(\theta^j)(W^j_E - W^j_U), \tag{15b}
\end{align*}
\]

where \( W^j_E \) denote the present discounted value of employment in sector \( j \) and \( W^j_U \) the present discounted value of unemployment. These equations have the same interpretation as the firm’s asset equations \( 15a \) and \( 15b \). Note that in equation \( 15b \), it is assumed, for analytical convenience, that unemployment insurance payments are null.

Wages are bargained at the sectoral level and are determined as the outcome of a Nash bargaining process between workers and employees where the value of a job for an entrepreneur is given by \( J^j_j \) in equation \( 13 \) and the surplus of a worker from a job is \( (W^j_E - W^j_U) \). Nash bargaining satisfies the first-order maximization condition \( (1 - \chi)(W^j_E - W^j_U) = \chi J^j_j \), where \( \chi \in [0, 1] \) denotes the bargaining power of workers.\(^{18} \) By substituting out \( J^j_j, W^j_E, \) and \( W^j_U \) in the latter condition, we can derive the expression characterizing the negotiated wage in each sector:

\[
w^j = \chi(p^j A^j + \delta^j \theta^j). \tag{16}
\]

To the extent that workers have some bargaining power \( (\chi > 0) \), the negotiated wage consists of a fraction of the firm’s surplus that accrues to the worker. This surplus includes the marginal product of labor, \( p^j A^j \), and the average search cost for each unemployed worker, \( \delta^j \theta^j \). This term reflects the influence of labor market conditions on the bargained wage and is increasing in \( \theta^j \).

In addition to the optimality conditions represented by \( 14 \) and \( 16 \), an equilibrium for the labor market is characterized by unemployed workers perfectly mobility across sectors. Indeed, a job seeker

\(^{18}\)To ensure the existence of a nontrivial equilibrium, we assume that \( \chi \) is strictly positive (otherwise a worker would never agree to the proposed wage) and strictly less than one (otherwise no firm will have an incentive to open a job).
should be indifferent between searching in either sector such that the asset value from search is equal across sectors. Specifically, the equilibrium has to satisfy $r^* W^T_U = r^* W^N_U$, which implies that
\[ \delta^T \theta^T = \delta^N \theta^N. \] (17)

As long as the costs of posting vacancies differ across sectors, differences in labor market tightness arise: the market tightness is lower and unemployment higher in the higher vacancy cost sector.

The search equilibrium is a collection of endogenous variables \{\hat{p}, \theta^T, \theta^N, w, w^N\} such that: (i) job creation satisfies (14) in each sector, (ii) wages are given by (16) in each sector and (iii) the no arbitrage condition (17) holds.\(^{19}\)

### 3.2.2 The Balassa-Samuelson effect with search frictions

In this section, we examine the model’s implications regarding the effects of biased productivity shocks favoring the traded sector. More precisely, we illustrate the extent to which the model’s predictions depart from those of the standard BS framework, and the role of the labor market frictions in accounting for this departure. To this end, the full set of equilibrium conditions (14), (16) and (17) are log-linearized around the steady-state. Denoting by $\alpha^N \in [0,1]$ the labor’s share of income in the non traded goods sector ($\alpha^N \equiv (w^N L^N)/(p Y^N)$), log-linearization yields

\[ \hat{p} = \left[ 1 - \xi \left( \frac{1 - \alpha^N}{1 - \chi} \right) \right] (\hat{A}^T - \hat{A}^N), \] (18a)

\[ \hat{w} = -\xi \left( \frac{\chi}{1 - \chi} \right) \left( \frac{1 - \alpha^N}{\alpha^N} \right) (\hat{A}^T - \hat{A}^N), \] (18b)

which highlights the dependence of both the relative price elasticity $\zeta_p$ and the relative wage elasticity $\zeta_w$ to the labor market rigidity inefficiency resulting from the search externalities.\(^{20}\) When search costs disappear ($\delta^T = \delta^N = 0$ so that $\alpha^N = 1$), the general equilibrium with labor market frictions tends to the Walrasian one: the wage equalization and the proportionality hypotheses are satisfied, so that $\zeta_p = 1$ and $\zeta_w = 0$. When labor market frictions are present ($\delta^T, \delta^N \neq 0$), equations (18a) and (18b) give the immediate results that $\zeta_p \in [0,1]$ and $\zeta_w \in [-1,0]$. Thus, the relative price increases less than one-for-one with productivity while the relative wage falls unambiguously. Both observations are consistent with our empirical evidence. Proposition 2 summarizes the main theoretical results.

**Proposition 2** Consider the two-sector model with search frictions. The elasticity of the relative price of nontradables and the relative wage with respect to the ratio of relative productivity are denoted by $\zeta_p$ and $\zeta_w$ and are given by (18a) and (18b) respectively. For any $\xi, \chi \in [0,1]$:

(i) $0 < \zeta_p < 1$,

(ii) $-1 < \zeta_w < 0$.

**Proof** Note that equation (16) for $j = N$ simplifies to $(\theta^N/p) = (\alpha^N/\chi) - 1$. The term $(\theta^N/p)$ being positive we obtain that $\alpha^N > \chi$. It follows that: $0 \leq (1 - \alpha^N)/(1 - \chi) \leq 1$ which, in turn, implies

\(^{19}\)Details of dynamics, the household maximization program and the steady-state equilibrium are laid out in an unpublished Appendix which is available on request from the authors.

\(^{20}\)The derivation of equations (18a) and (18b) is depicted in Appendix B. The term $(1 - \alpha^N) \in [0,1]$ is the sum of the shares of profits ($\alpha^N_H$) and vacancy costs ($\alpha^N_V$) in the income generated in the non traded goods sector according to $1 - \alpha^N = \alpha^N_H + \alpha^N_V$, where $\alpha^N_H \equiv [(r^N L^N)/(p Y^N)]$ and $\alpha^N_V \equiv (\delta^N V^N)/(p Y^N)$. 
that \(0 \leq 1 - \xi(1 - \alpha^N)/(1 - \chi) \leq 1\) as \(\xi \in [0, 1]\). This establishes formally item (i) of Proposition 2. By combining the inequality \((\chi/\alpha^T) < 1\) with \(0 \leq (1 - \alpha^N)/(1 - \chi) \leq 1\) and the fact that \(\xi \in [0, 1]\), we obtain \(-1 \leq -\xi[\chi/\alpha^N][(1 - \alpha^N)/(1 - \chi)] \leq 0\). This states item (ii) of Proposition 2. ■

Proposition 2 makes clear the central role of labor market frictions in the adjustment of the relative price and sectoral wages in response to productivity shocks. Our specification of the search parameters makes the equilibrium labor market tightness in the traded sector \(\hat{\theta}^T\) independent of productivity. Combining (14) with (16) results in a constant labor market tightness given by the solution to

\[
1 - \chi(1 + \hat{\theta}^T) - \frac{(r^* + \lambda^T)}{q(\hat{\theta}^T)} = 0.
\] (19)

According to (19), the equilibrium labor market tightness in the traded sector is invariant to sectoral productivity shocks. The source of this invariance comes from our assumption that the vacancy cost is perfectly indexed to productivity, leading to a one-for-one increase in the profit from job creation in sector \(T\) \((J^T_U = \hat{A}^T)\), and resulting in a constant ratio of vacancies to unemployment \(\hat{\theta}^T\). With an unchanged labor market tightness in the traded sector, both the match surplus that accrues to firms and workers’ part of the surplus increase by the same quantity \((\hat{W}_E^T - \hat{W}_U^T = \hat{J}_T^T = \hat{A}^T)\). With bargaining power held constant, the negotiated wage \(\hat{w}^T\) fully absorbs the productivity change \((\hat{w}^T = \hat{A}^T)\). In the non traded sector, there is an increase in the profitability of jobs too, thus the wage \(\hat{w}^N\) and market tightness \(\hat{\theta}^N\) are pushed up in that sector. However, from the log-linearized version of the model, it can be shown analytically that wages in sector \(N\) rises, but, by less than in the traded sector \((\hat{w}^N < \hat{w}^T)\) and, as item (ii) of Proposition 2 shows, the relative wage falls. The intuition is that if the economy experiences biased productivity growth favoring the sector \(T\), wages \(\hat{w}^T\) and the expected utility of an unemployed person \(\hat{W}_U^T\) rise. Perfect mobility of unemployed persons between sectors induces unemployed workers from the non traded sector to search for jobs in the traded sector. This process continues until the present discounted value of unemployment are equalized again \(\hat{W}_U^T = \hat{W}_U^N\). Consequently, from (17), this reallocation effect induces an increase in the labor market tightness of the non traded sector. As unemployed workers in sector \(N\) benefit also from the productivity improvement through the increase in \(\hat{w}^N\), the negotiated wage \(\hat{w}^N\) does not fully absorb the technological shock. Hence, the wage increase in sector \(N\) is lower to that predicted by the BS model \((\hat{w}^N < \hat{A}^T = \hat{w}^T)\).

The first part of Proposition 2 states that a rise in the relative productivity of tradables does not imply a proportional relative price of nontradables increase. The quantitative impact of sectoral productivity shocks on \(\hat{p}\) requires the log-linearization of (14) and (16), which yields:

\[
\hat{p} = \alpha^N(\hat{w}^N - \hat{A}^N) + (1 - \alpha^N)(1 - \xi)\hat{\theta}^N.
\] (20)

It follows that the relative price change is a weighted average of the change in marginal costs \((\hat{w}^N - \hat{A}^N)\) and the change in the labor market tightness in nontradables \((\hat{\theta}^N)\), with the weights being a function of \(\alpha^N\), the labor share of production in the non traded good sector and the elasticity of the matching function with respect to unemployment \((1 - \xi)\). The higher the labor market frictions (i.e. the lower is \(\alpha^N\)), the weaker the effect of the change in the marginal costs and the stronger the effect of the change in labor market tightness.\(^{21}\) Thus, the relative price response to productivity shock depends positively

\(^{21}\)The standard BS effect, operative in the frictionless case, is obtained by setting \(\alpha^N = 1\) (search costs are null) and \(\hat{w}^N = \hat{A}^T\) (wage equalize across sectors) in equation (20). In this situation, the relative price increase stems only from higher marginal costs.
on the change in labor market tightness: a tighter labor market increases total costs, which, in turn, affects positively nontradables inflation. The reason for this is that higher labor market frictions in sector $N$ act like a negative productivity shock in this sector, which increases non traded firms’ costs. As a result, the equilibrium value of the relative price rises allowing non traded goods producers to cover higher production costs. In sum, high labor market rigidities are good for nontradables inflation because with a large share of search costs in production, the relative price is partially immunized from shocks to productivity. Thus, a country with high labor market rigidities may enjoy the benefits of technological progress without experiencing a strong internal price inflation (but at some cost, the relative wage drops).

Simulations are conducted to verify whether the model can replicate quantitatively the empirical facts, with results reported in Table 4. The unit of time is a year. The real interest rate in the economy $r^*$ is set to 4%. The sectors’ productivity parameters are normalized to $A^T = 1.5$ and $A^N = 1$ respectively. Consistent with the Ghironi and Melitz’s [2005] parametrization, our calibration implies that traded firms are 50% more productive than non traded firms. We normalize the parameters related to vacancy costs $\gamma^T$ and $\gamma^N$ to one, but we point out that the quantitative results do not depend on the values of these parameters. The (annual) exogenous separation rates $\lambda^T$ and $\lambda^N$ are set to match the US evidence in Davis et al. [2010] who find a job destruction rate of 12% for the manufacturing industry and 20% for the services sector. Consider those industries as reasonable proxies of the tradable and nontradable sectors respectively, we set $\lambda^T = 0.12$ and $\lambda^N = 0.20$. The elasticity of matches with respect to vacancies is set to $\xi = 0.5$, which is in the range of reasonable values suggested by Petrongolo and Pissarides [2001]. Given the lack of direct empirical evidence on the bargaining power, the parameter $\chi$ is set to a conventional value of 0.5 for reasons of symmetry. The equality between $\xi$ and $\chi$ ensures the efficiency of decentralized equilibrium (Hosios [1990]). Imposing the Hosios and symmetry conditions, it follows, from (18a) and (18b), that the relative price elasticity to sectoral technological shocks $\zeta_p$ reduces to $\alpha^N$ and the relative wage elasticity $\zeta_w$ is equal to $-0.5(1 - \alpha^N)/\alpha^N$. Thus, the critical parameter for $\zeta_p$ and $\zeta_w$ is $\alpha^N$, the labor share of production in the non traded good sector. Given the prominent role of this parameter, a sensitivity analysis is carried out by taking different values of $\alpha^N$. Over the period 1980-2007, the labor share in nontradables ranges from 0.63 to 0.82 across OECD countries (with an average of 0.70). However, the labor share of production in sector $N$ is not exogenous in the model, but rather endogenously determined by the steady-state equilibrium. As the model is designed to replicate a typical OECD economy, we choose the matching efficiency in sector $N$, $m^N$, to reflect a labor share $\alpha^N$ in the range given above. To obtain $\alpha^N = \{0.6, 0.7, 0.8\}$, one has to set $m^N = \{0.36, 0.965, 2.17\}$.

Table 4 presents the results of this exercise and shows that the BS augmented with matching frictions predicts both an increase in the relative price and a drop in the relative wage in response to the rise in the relative productivity that are similar in magnitude to those estimated using actual data. For the most empirical relevant situation, $\alpha^N = 0.70$ according to our estimates, the theoretical elasticity for the relative price is 0.70 and -0.21 for the relative wage, values close to our empirical estimates.

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22 Own calculations based on data from KLEMS (see Appendix A for details).
23 The matching efficiency parameter in tradables $m^T$ is set to 1.3 to reproduce an unemployment rate of around 12% in the US manufacturing sector (source: BLS).
Table 4: Responses of $\tilde{p}$ and $\tilde{w}$ to changes in $A_T/A_N$ (search model)

<table>
<thead>
<tr>
<th>$\alpha_N$</th>
<th>$\zeta_p$</th>
<th>$\zeta_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.60</td>
<td>-0.33</td>
</tr>
<tr>
<td>0.7</td>
<td>0.70</td>
<td>-0.21</td>
</tr>
<tr>
<td>0.8</td>
<td>0.80</td>
<td>-0.13</td>
</tr>
<tr>
<td>BS model ($\alpha_N = 1$)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Empirical estimates</td>
<td>$\hat{\beta}_p = 0.68$</td>
<td>$\hat{\beta}_w = -0.24$</td>
</tr>
</tbody>
</table>

4 Conclusion

In this paper we document the effects of sectoral productivity differentials on the relative price of non-tradables and the relative wage for a panel of fourteen OECD countries. We find that the transmission of productivity shocks is likely to differ from that of the Balassa-Samuelson model since we strongly reject both the wage equalization and proportionality implications of this framework. We argue that one should use a model with labor market frictions to understand properly the transmission mechanisms of technological shocks. To do this, we construct and simulate two versions of a dynamic general equilibrium two-sector model which feature labor reallocation costs or matching frictions.

Introducing these frictions affects the response of both the relative price and the relative wage to sectoral productivity shocks. In particular, the relative price increases less than one-for-one with productivity whereas the relative wage falls unambiguously, results that are consistent with our empirical evidence. Second, the strength of labor market frictions matters significantly. In particular, the higher the inefficiencies the lower the relative price responsiveness to sectoral productivity shocks and the larger the relative wage drops. Hence, a country with an high degree of labor market frictions may enjoy the benefits of technological progress without experiencing an excessive rise in its internal prices.

A Appendix: data construction and cointegration tests results

We consider annual data taken from the sectoral KLEMS database. Data covers a maximum period from 1970 through 2007, for a total of fourteen industrialized countries and eleven industries. The country sample consists of Belgium, Denmark, Spain, Finland, France, Germany, Ireland, Italy, Korea, Japan, Netherlands, Sweden, the UK and the US. Following De Gregorio et al. [1994], Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport, Storage and Communication; and Financial Intermediation are classified as traded goods. Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Real Estate, Renting and Business Services; and Community Social and Personal Services account for the non traded sector. KLEMS database contains data on value added in current and constant prices, labor compensation and employment for each sector, permitting the construction of sectoral value-added deflators and wages, and the derivation of sectoral labor productivity indexes.

In what follows, subscript $i$ refers to country, $k$ industry, $t$ time and $j$ sector ($j = T, N$). Define value-added measured at current prices ($VA$) and value-added volume ($VAV$) for sector $j$, country $i$ as $VA^j_{i,t} = \sum_{k \in j} VA^j_{k,i,t}$ and $VAV^j_{i,t} = \sum_{k \in j} VAV^j_{k,i,t}$, where $VA^j_{k,i,t}$ ($VAV^j_{k,i,t}$ resp.) is the value-added measured at current prices (value-added volume resp.) for industry $k$ classified in sector.
j. All prices are value-added deflators: \( p^{i,t}_j = VAV^j_{i,t}/VAV^j_{i,t} \). The relative price of non traded goods for country \( i \) is therefore defined by \( p^{i,t}_j = p^N_{i,t}/p^T_{i,t} \).

Labor productivity or real output per worker index for sector \( j \) is given by \( A^j_{i,t} = VAV^j_{i,t}/L^j_{i,t} \), where total employment or equivalently persons engaged (\( L^j_{i,t} \)) used in sector \( j \), country \( i \) at time \( t \) is given by \( L^j_{i,t} = \sum_{k \in j} L^j_{k,i,t} \) where \( L^j_{k,i,t} \) is total employment in industry \( k \) classified in sector \( j \), country \( i \) at time \( t \).

Wage data for sector \( j \) are constructed by dividing labor compensation by employment in that sector, yielding the wage per worker: \( w^{i,t}_j = COMP^j_{i,t}/L^j_{i,t} \), where total employment or equivalently persons engaged (\( L^j_{i,t} \)) used in sector \( j \), country \( i \) at time \( t \) is given by \( L^j_{i,t} = \sum_{k \in j} COMP^j_{k,i,t} \) where \( COMP^j_{k,i,t} \) is labor compensation in industry \( k \) classified in sector \( j \), country \( i \) at time \( t \).

Table 5 contains the results of Pedroni’s ([1999], [2004]) cointegration tests based on the estimated residuals of equations (1a) and (1b).

<table>
<thead>
<tr>
<th>Table 5: Panel cointegration tests results (p-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>eq. (1a)</td>
</tr>
<tr>
<td>Panel tests</td>
</tr>
<tr>
<td>Non-parametric ( \nu )</td>
</tr>
<tr>
<td>Non-parametric ( \rho )</td>
</tr>
<tr>
<td>Non-parametric ( t )</td>
</tr>
<tr>
<td>Parametric ( t )</td>
</tr>
<tr>
<td>Group-mean tests</td>
</tr>
<tr>
<td>Non-parametric ( \nu )</td>
</tr>
<tr>
<td>Non-parametric ( t )</td>
</tr>
<tr>
<td>Parametric ( t )</td>
</tr>
</tbody>
</table>

**B The model with search frictions: steady-state variations**

Differentiating (14), (16) and (17) with respect to \( \tilde{p} \), \( \tilde{\theta}_T \) and \( \tilde{w}_N \) for \( j = T, N \) yields the following system of equations:

\[
\begin{bmatrix}
0 & a_{12} & 0 & 1 & 0 \\
-A^T & 0 & a_{23} & 0 & 1 \\
0 & -\chi \delta^T & 0 & 1 & 0 \\
-\chi A^N & 0 & -\chi \delta^N & 0 & 1 \\
0 & \delta^T & -\delta^N & 0 & 0
\end{bmatrix}
\begin{bmatrix}
d\tilde{p} \\
d\tilde{\theta}_T \\
d\tilde{\theta}_N \\
d\tilde{w}_T \\
d\tilde{w}_N
\end{bmatrix}
= \begin{bmatrix}
\alpha^T & 0 \\
\tilde{\theta}_T \chi (1 + \tilde{\theta}_T) & 0 \\
\tilde{\theta}_N \chi (1 + \tilde{\theta}_N) & 0 \\
\tilde{\theta}_T & \tilde{\theta}_N
\end{bmatrix}
\begin{bmatrix}
dA^T \\
dA^N
\end{bmatrix},
\]

where the terms \( a_{12} \) and \( a_{23} \) are given by

\[
a_{12} = \frac{(A^T - \tilde{w}_N^T)(1 - \xi)}{\tilde{\theta}_T} > 0, \quad a_{23} = \frac{(\tilde{p}A^N - \tilde{w}_N^N)(1 - \xi)}{\tilde{\theta}_N} > 0. \quad (B2)
\]

The derivatives for \( \tilde{p} \) can be obtained by the standard procedure of the Cramer rule:

\[
\frac{d\tilde{p}}{dA^T} \frac{A^T}{\tilde{p}} = -\frac{d\tilde{p}}{dA^N} \frac{A^N}{\tilde{p}} = \left[ 1 - \xi \left( \frac{1 - \alpha^N}{1 - \chi} \right) \right] > 0.
\]
Equation (18a) in the text derives from (B3). Variations in sectoral labor tightness are given by:

\[
\frac{d\tilde{\theta}^T}{dA^T} = 0, \hspace{1cm} \frac{d\tilde{\theta}^N}{dA^T} = 1. \tag{B4a}
\]

The following expression for the steady-state deviation of \(\tilde{w}^T\) and \(\tilde{w}^N\) can be also derived in the form:

\[
\frac{d\hat{\tilde{\omega}}^T}{dA^T} = \hat{\tilde{w}}^T > 0 \quad \text{and} \quad \frac{d\hat{\tilde{\omega}}^N}{dA^N} = 0, \tag{B5a}
\]

\[
\frac{d\hat{\tilde{\omega}}^N}{dA^T} = \hat{\tilde{w}}^N \left[ 1 - \xi \left( \frac{\chi}{1 - \chi} \right) \left( \frac{1 - \alpha^N}{\alpha^N} \right) \right] > 0, \tag{B5b}
\]

\[
\frac{d\hat{\tilde{\omega}}^N}{dA^N} = \hat{\tilde{w}}^N \xi \left( \frac{\chi}{1 - \chi} \right) \left( \frac{1 - \alpha^N}{\alpha^N} \right) > 0. \tag{B5c}
\]

Finally, combining (B5a), (B5b) and (B5c) leads to equation (18b) in the text.

References


Garcia-Cebro J.A. and R. Varela-Santamaria (2009), “Imperfect Intersectoral Labor Mobility and Monetary Shocks in a Small Open Economy”, Open Economics Review.


