Inflation, debt enforcement and currency competition*

Mariana Rojas Breu†

September 2010

Abstract

International, low-inflation, currencies are increasingly available everywhere. However, domestic currencies remain the means of payment mostly used in the vast majority of countries. This observation conflicts with the literature on currency competition which predicts that, in absence of transaction costs, agents will prefer to use the less inflationary currency. In this paper, I provide a framework in which inflationary currencies are used, despite the availability of a less inflationary currency. I suggest that the use of domestic currencies is associated with the legal environment that determines the degree of debt enforcement. My key assumption is that full enforcement is not feasible. This entails that the rate of return on currencies is in part determined by incentives eliciting voluntary debt repayment. I show that, under certain conditions, the inflation rate of a currency in which debts are denominated may function as a commitment device. As a result, the more inflationary currency is preferred in equilibrium, despite the absence of costs in using the less inflationary currency.

Keywords: Money, Currency Competition, Debt Enforcement, Banking

JEL Classification: E41, E50, E51

---

*I would like to thank Jung-Hyun Ahn, David Andolfatto, Luis Araujo, Jean-Pascal Bénassy, Aleksander Berentsen, Vincent Bignon, Régis Breton, Jean Cartelier, Carlos Garriga, Wouter den Haan, Hubert Kempf, Antoine Martin, Cyril Monnet, Guillaume Rocheteau, Martin Schneider, Christopher Waller, Warren Weber and Randall Wright for very helpful discussions, as well as participants at Federal Reserve Bank of Chicago, Federal Reserve Bank of Philadelphia, Federal Reserve Bank of Saint Louis, Michigan State University, Banque de France, Erasmus School of Economics, Université Paris Ouest, Universität Basel and European Macroeconomics Workshop. I acknowledge financial support from the Swiss National Science Foundation. Any errors are my own.

†Mariana Rojas Breu: Swiss National Science Foundation & Visiting Researcher at Erasmus School of Economics. Phone: +31 (0)10 408 1425, Fax: +31 (0)10 408 9161, E-mail: mariana.rojas-breu@unibas.ch
1 Introduction

International currencies, characterized by low inflation rates, are easily available in most economies. However, domestic currencies remain the means of payment predominantly used in the vast majority of countries. This observation is puzzling and conflicts with the literature on currency competition which predicts that, in absence of transaction costs, agents will prefer a less inflationary currency to hold and to accept in trade. In this paper, we provide a framework in which inflationary currencies are used, despite the existence of less inflationary currencies and despite the absence of costs in using them. That is, we establish conditions under which the more inflationary currency may be effectively preferred to the less inflationary currency.

To provide a rationale for the use of domestic currencies, we propose an avenue different from the one that assumes a cost to the use of one or both of the competing currencies. We suggest that the use of domestic currencies is associated with the technology available for enforcing borrowers. We consider the issuance of credit backed by deposits, in an environment where full enforcement is not feasible. This entails that the rate of return on currencies is in part determined by incentives eliciting voluntary debt repayment. We show that, under certain conditions, the inflation rate of a currency in which debts are denominated may function as a commitment device.

The mechanism can be described as follows. Depending on the enforcement technology available to the institutions which provide credit, that for simplicity we call banks, the currency in which debts are denominated may affect the outside option for defaulters. In particular, if banks are only able to enforce agents who carry out transactions in the market and are only able to do so temporarily, then the punishment on defaulters is stronger if the currency in which they took out loans is more inflationary. The reason is that defaulters would choose not to participate in the market for a while in order to avoid being enforced by banks. Doing this would entail a lower benefit, the more inflationary the currency borrowed were, since the currency would be less valuable at the moment to use it to purchase goods. Thus, borrowing a more inflationary currency would be a better commitment device than borrowing a less inflationary currency. In a rational-expectations framework, depositors anticipate that borrowers can commit better to paying interest on loans denominated in the more inflationary currency. Hence, agents prefer to carry the more inflationary currency across periods since this provides a higher compensation for their deposits.

We refer to the technology described in the previous paragraph as imperfect enforcement, since it enables banks to enforce some agents (those who take part in the market) but not all of them and, further, banks can enforce agents only for some time. To highlight the assumptions that give rise to the preference for the more inflationary currency when enforcement is imperfect, we also consider the case of a perfect enforcement technology.
and the case where no enforcement is feasible. We show that, even though agents may prefer the more inflationary currency when enforcement is imperfect, they never do so when enforcement is perfect or not feasible at all. The reason is that, with perfect enforcement as well as with no enforcement, agents do not get a punishment that depends on the currency that is borrowed. With perfect enforcement, they get no punishment since default is not an option. With no enforcement, they are free to use the funds of the defaulted loan in the same period of the loan; consequently, inflation does not affect its value, and borrowing in a more inflationary currency does not make agents less willing to default, which is contrary to the case with imperfect enforcement.

Our result may seem contradictory with what Kareken and Wallace (1981) defined as the dominance result. According to the dominance result, if there are two fiat currencies with positive price and no transaction costs or legal restrictions, their inflation rates must be equal. The reason is that if one of the currencies had a higher inflation rate, in equilibrium it would be driven out by the other currency so that its price would be zero. The explanation to this apparent contradiction is that, while in Kareken and Wallace inflation only determines the rate at which money depreciates, in our model it also affects the outside option for defaulters. Therefore, two currencies may have the same rate of return, despite their inflation rates being different, as long as the interest rate in the higher-inflation currency is sufficiently higher than the interest rate in the lower-inflation currency. As we will show, this may occur when inflation affects the incentives to default.

In order to put in place the mechanism described, we develop a Lagos and Wright (2005) model where agents can make deposits and take out loans, as in Berentsen, Camera and Waller (2007). The first difference with Berentsen et al. is that we consider an environment with two currencies, instead of one. Agents are allowed to borrow and deposit in either of the currencies. Thus, the only feature that potentially makes currencies differ in our model is their inflation rates. Another difference with respect to Berentsen et al. is that we add the imperfect enforcement set-up, which is key to letting incentives to default depend on the currency denomination of loans.

After the dominance result mentioned above, the literature which addresses the competition between currencies has mainly relied on legal restrictions or transaction costs to define the existence of national-currencies equilibria. Transaction costs and legal restrictions allow low-return currencies to circulate in equilibrium despite the existence of a competing, higher-return currency. For example, Matsui (1998) studies different equilibria in a two-country model where taxes must be paid in local currency, the government injects local currency through purchases of goods and currency exchange is costly. Martin (2006) studies a cash-in-advance economy where there is a cost for sellers to accept two currencies and a proportional cost of trading on the currency-exchange market. The national currency may
circulate in equilibrium unless its growth rate is too high. Engineer (2000) considers a de-
centralized economy where the domestic currency has lower transaction costs but a higher
growth rate than the foreign currency.

Several papers that belong to the so-called first and second generations of the search-
theoretic approach have proved fruitful in studying national-currencies equilibria. A first
group of papers study currency competition by assuming that there is no ex-ante difference
in the fiat monies that compete, except for the physical properties that make them distin-
guishable (for instance, Matsuyama, Kiyotaki and Matsui (1993), Shi (1995) and Trejos and
Wright (1996)). Hence, these papers focus on the role of expectations for a currency to be
accepted in trade and emphasize the multiplicity of equilibria when it comes to studying the
acceptability of money. Thus, this literature does not aim to provide an explanation for the
use of domestic currencies but rather to explain why its use is one of the possible equilibrium
outcomes.

A second group of search-theoretic models assume an exogenous difference in currencies’
returns. Since in this class of models money is indivisible, the difference in return is modeled
through the behavior of a fraction of agents, taken as exogenous. In Li and Wright (1998), a
proportion of agents are government agents who are assumed to use the domestic currency. In
Curtis and Waller (2000), government agents impose a fine with some probability on traders
who use the foreign (illegal) currency, whereas in Curtis and Waller (2003) and Camera, Craig
and Waller (2004) they may confiscate money holdings in their meetings with private agents.
That framework made it possible to have two currencies with different returns coexisting
and avoid the dominance result because in a search-environment arbitrage opportunities are
reduced and because of the indivisibility of money. This literature provided insight on the
features that allow for either one or both of the currencies to be used in equilibrium and on
the equilibrium properties in terms of purchasing power of the monies. However, once one
attempts to analyze the effect of money growth on currencies’ choice -which, for instance,
the divisible money set-up by Lagos and Wright allows for-, obtaining one equilibrium in
which the more inflationary currency circulates without assuming costs to the use of the
competing currency is not straightforward. The purpose of this paper is to suggest the issue
of default on debts as a key feature to give rise to such equilibrium.

Our work is also related to the literature on the optimal inflation rate. In particular,
our work builds on the literature that states a benefit from inflation owing to its effect on
incentives to default. Berentsen et al. develop a Lagos and Wright model with banks that
provide loans and take deposits to analyze how inflation affects welfare. They show that
when no enforcement on borrowers is feasible, inflation may be welfare improving because it
makes the outside option for defaulters less attractive. As a result, inflation may allow for
an increase in the borrowing interest rate without promoting default by borrowers, which
in turn increases the value of money and so the goods that money can purchase. Since consumption is increasing in inflation, inflation has a positive impact on welfare. Previously, Aiyagari and Williamson (2000) have presented computational results on the benefits of inflation arising from the role of inflation in increasing the punishment on defaulters. This argument is also studied in Antinolfi, Azariadis and Bullard (2007) and Diaz and Perera-Tallo (2008).\footnote{Other features of the environment have been highlighted to play a role in the welfare benefits of inflation. For instance, it has been argued that inflation can improve welfare in environments where search effort is endogenous (see Berentsen, Rocheteau and Shi (2007)) or that it can reduce the level of socially costly cash activities such as theft (see He, Huang and Wright (2008)).} We aim to contribute to this literature by studying how the results are affected when defaulters have a competing currency as an outside option. Indeed, we will see that conclusions obtained with one currency may not hold when a second currency is introduced. Furthermore, whereas the literature on the optimal inflation rate examines the conditions under which inflation turns out to be beneficial from a social planner’s perspective, our purpose is to identify conditions under which agents may choose to use the more inflationary currency.

In the next section we present the model. In section 3, we characterize the symmetric equilibrium. In section 4, we focus on the national-currency equilibrium and study the tree different set-ups concerning the enforcement technology mentioned above. Finally, section 5 concludes.

2 Environment

The original framework we build on is the divisible money model by Lagos and Wright (2005). The main advantage of this framework is that it facilitates the introduction of heterogeneity in production and consumption preferences as well as the divisibility of money, keeping the distribution of money holdings degenerate and, thus, analytically tractable. More precisely, we base our model on the model developed by Berentsen et al. (2007).

Time is discrete and goes for ever. There is a continuum of infinitely-lived agents of unit mass and two types of perfectly divisible and non-storable goods: a market-good and a home-made good. Agents can only consume the home-made good produced by themselves and cannot consume the market good produced by themselves. They discount across periods with factor $\beta \in (0, 1)$.

In each period, two competitive markets open sequentially. Before the first market opens, agents get an idiosyncratic preference shock by which they learn that they get utility from consumption and cannot produce the market-good (with probability $(1 - s)$) or that they get no utility from consumption but can produce the market-good (with probability $s$). We
call consumers the agents who get the first type of shock and sellers those who get the second type.

After the first shock, a second shock occurs which affects only consumers. With probability \( (1 - b - s) / (1 - s) \), each consumer learns that he only gets utility from the home-made good. With probability \( b / (1 - s) \), each consumer learns instead that he only gets utility from the market-good. Consumers who prefer the home-made good are home-consumers and consumers who prefer the market good are buyers.

In the first market, buyers get utility \( u(q) \) when they consume a quantity \( q \) of any of the market good, with \( u'(q) > 0, u''(q) < 0, u'(0) = +\infty \) and \( u'(\infty) = 0 \). Producing a quantity \( q \) represents a disutility equal to \( c(q) = q \). We do not explicitly model the choice of home-consumers; instead we assume that they get some fixed net utility from consuming and producing their home-made good.

In the second market, all agents may consume and produce the market-good. Home-consumers may, in addition, consume their home-made good, and they get the same utility from a unit of their home-made good as from a unit of a market good. Consuming \( x \) gives utility \( U(x) = \ln(x) \).\(^2\) Disutility from producing \( x \) is equal to \( y \), where one unit of labor yields one unit of good.

In addition, two different monies exist. We call them national currency and foreign currency. Both are intrinsically useless. We assume that they are issued by a national central bank and a foreign central bank, respectively, even though we analyze only one country. Indeed, what is important for our purpose is that two different currencies with potentially different inflation rates are available. We call \( M_{t,n} \) and \( M_{t,f} \) the money stocks of national currency and foreign currency, respectively, in period \( t \).

Central banks’ decisions are exogenous. The growth rates of national and foreign currency are \( \gamma_n \) and \( \gamma_f \) with \( \gamma_n, \gamma_f > 0 \), so that \( M_{t,n} = \gamma_n M_{-1,n} \) and \( M_{t,f} = \gamma_f M_{-1,f} \), where the subscript \(-1\) indicates the previous period \((+1\) indicates the following period\). Agents receive lump-sum transfers from each authority equal to \((\gamma_n - 1) M_{-1,n}\) and \((\gamma_f - 1) M_{-1,f}\) at the beginning of the second market in period \( t \).\(^3\) Currencies may be converted at no cost during the second market.

Agents can deposit and borrow money (national as well as foreign currency) by resorting

\(^2\)The assumption on a logarithmic utility function in the second subperiod is a sufficient although not necessary condition for the results that follow. What is actually necessary is that \( U(x^*) - x^* \), where \( x^* \) is determined by \( U''(x^*) = 1 \), is sufficiently high, or that \( U(0) \) is sufficiently low.

\(^3\)The assumption on national agents receiving transfers in national currency and foreign currency may seem odd. However, the purpose of it is to ensure symmetry between currencies, except potentially for their money growth rates, in order to highlight the role of enforcement in determining the existence of national-currency equilibria (instead of other features of the environment that could cause the national currency to be used).
to banks. Banking activities take place after the first preference shock and before the second one. Banks are competitive and face an exogenous level \( r \) of reserve requirements; i.e., they can only issue a total amount of loans \( L_j \) by keeping a ratio \( r = D_j/L_j \) of total deposits to total loans, where \( j = n, f \) indicates national or foreign currency and \( 0 \leq r \leq 1 \).

Loans are issued as bilateral contracts between an agent and a bank by which the bank gives an amount of money to the agent and the agent must pay it back during the second market together with the interest on it. Deposits are taken by banks and paid back during the second subperiod with the corresponding interest. The timing of events is depicted in Figure 1.

\[
\begin{array}{ccc}
 & t - 1 & t & t + 1 \\
1^{st} \text{market} & 1^{st} \text{market} & 2^{nd} \text{market} & 2^{nd} \text{market} \\
1^{st} \text{shock} \rightarrow \text{Banks} \rightarrow 2^{nd} \text{shock} \rightarrow \text{Trade} & \text{Trade} & \text{Trade} \\
& \text{Repayment} & \text{Repayment} \\
& \text{Currency Exchange} & \text{Currency Exchange} \\
& \text{Transfers} & \text{Transfers} \\
\end{array}
\]

Figure 1: Timing of events

We will analyze three different set-ups regarding the technology for enforcing loans repayment: perfect enforcement, no enforcement and imperfect enforcement. Perfect enforcement means that banks are able to force agents to work to repay their loans and the interest. By contrast, no enforcement means that banks are unable to enforce borrowers and, therefore, must establish conditions so that repayment is voluntary. By imperfect enforcement, we mean that banks have an enforcement technology at their disposal by which they can force repayment by those agents who enter the second market, in which settlement of debts takes place. However, they cannot enforce agents who decide not to enter the second market. In addition, when enforcement is imperfect banks can enforce agents who participate in the second market only in the period in which they have borrowed, but they are not able to do so afterwards.

In the cases of no enforcement and imperfect enforcement, we assume that banks possess a record keeping technology that allows to punish defaulters by excluding them from the banking system for the rest of their lifetime; i.e., after defaulting, agents are prevented from borrowing and depositing.\(^4\) Moreover, we assume that defaulters are excluded from the monetary transfers as well.

\(^4\)We follow Berentsen et al. who make this assumption in order to study the effect of inflation on welfare when enforcement is not feasible. We depart from their set-up in that we introduce the case of imperfect enforcement.
Banks operate at a cost \( \kappa \) per unit of money loaned. This cost is aimed to capture the resources devoted to the enforcement technology and the record keeping on borrowers.

In order to motivate a role for money, we assume anonymity of traders so that, for trade to take place, sellers require compensation at the same time as they produce. This assumption rules out bilateral credit; however, it does not conflict with the existence of lending in this model because this only requires that agents are identified by banks (which is not the same as being identified by traders).

Dealing with two currencies requires additional precisions. The focus of our currency-choice analysis is on the currencies’ returns rather than on expectations (which has already been done in the literature, as mentioned in the introduction). In particular, our main purpose is to assess conditions under which agents prefer to use the domestic currency, even though a foreign currency is available. Thus, we assume that agents are free to move to another country, completely exogenous for our purpose, in which the foreign currency circulates and which is identical otherwise. Moreover, agents have access to banks in this other country, unless they default before moving. Agents may irreversibly move during the settlement stage. In the case of imperfect enforcement, a defaulter may move during the settlement stage of the period after defaulting, when banks have no enforcement power anymore.\(^5\) The existence of this alternative country as an outside option is meant to authorize agents to switch to the foreign currency, if they want to, without dealing with coordination issues. However, it does not give rise to any difference between currencies.\(^6\) In sum, neither transaction costs nor legal restrictions are associated to the use of either of the currencies.

3 Symmetric equilibrium

We look at symmetric and stationary equilibria. Imposing stationarity implies that end-of-period real money holdings are constant:

\[
\begin{align*}
\gamma_j &= \frac{M_{+1,j}}{M_j} \\
\phi_j M_j &= \phi_{+1,j} M_{+1,j}
\end{align*}
\]

where \( \phi_j \) is the price of currency \( j \) in real terms.

A representative agent starts each period with amounts \( m_n \) of national currency and \( m_f \) of foreign currency. His expected utility when each period starts is \( V(m_n, m_f) \). When an

\(^5\)Since agents receive transfers in both currencies, the decision to move is assumed to be irreversible to rule out moving by agents who only want to spend the transfers received in a currency different from the one used in the own country, which is not the focus of this paper.

\(^6\)We could also authorize agents to move to an identical country where the national currency circulates. However, this will not affect conditions for existence of national-currency equilibria.
agent enters the second market, his expected utility is $W(m_n, m_f, d_n, d_f, l_n, l_f)$, according to his money holdings, deposits and loans in each currency.

### 3.1 The second market

Agents decide whether to enter the second market or not. Buyers always decide to enter the second market since they can only consume the market-good produced by other agents and their consuming zero goods entails infinite negative utility. Instead, home-consumers may decide not to enter the second market, since they may consume their home-made good produced by themselves.

For agents who enter the second market, their program is to solve:

$$W(m_n, m_f, d_n, d_f, l_n, l_f) = \max_{x,y,m_{+1,j}} U(x) - y + \beta V_{+1}(m_{+1,n}, m_{+1,f})$$

subject to:

$$y = x + \sum_j \phi_j [m_{+1,j} - m_j - (1 + i_{d,j}) d_j + (1 + i_j) l_j - (\gamma_j - 1) M_{-1,j}]$$

Agents can hold deposits $d$, loans $l$ and money in national currency and/or foreign currency. $i_d$ is the deposit interest rate and $i$ is the borrowing interest rate. We rewrite (2) using the budget constraint:

$$W(m_n, m_f, d_n, d_f, l_n, l_f) = \max_{x,m_{+1,j}} U(x) - x + \beta V_{+1}(m_{+1,n}, m_{+1,f})$$

$$+ \sum_j \phi_j [m_j + (1 + i_{d,j}) d_j - (1 + i_j) l_j - m_{+1,j} + (\gamma_j - 1) M_{-1,j}]$$

The first-order conditions on $x$ and $m_{+1,j}$ are:

$$U'(x) = 1$$

$$\frac{\partial V_{+1}}{\partial m_{+1,j}} = \frac{\phi_j}{\beta}$$

The envelope conditions on $m_j$, $d_j$ and $l_j$ are:

$$W_{m,j} = \phi_j$$

$$W_{d,j} = \phi_j (1 + i_{d,j})$$

$$W_{l,j} = -\phi_j (1 + i_j)$$

Home-consumers who do not enter the second market simply maximize the difference between their utility from consumption $U(x)$ and their disutility from working $x$. However, they cannot adjust their money holdings since they do not trade.
3.2 The first market

3.2.1 Sellers

In the first market, sellers decide how much to produce in exchange for each currency, \( q^*_n \) and \( q^*_f \), and how much to deposit in each currency, \( d_n \) and \( d_f \). Their program is:

\[
\max_{q^*_j,d_j} \left[ q^*_j - W (m_{s,n}, m_{s,f}, d_n, d_f, 0, 0) \right] \\
\text{s.t. } d_j \leq m_{-1,j}, \quad m_{s,j} = m_{-1,j} - d_j + p_j q^*_j, \quad q^* = q^*_n + q^*_f
\]

where \( m_{s,n} \) (\( m_{s,f} \)) is the amount of national (foreign) currency that the seller brings into the second market. The first-order condition on \( q^*_j \) is:

\[
-1 + \phi_j p_j = 0
\]  

(5)

The first-order condition on \( d_j \) is:

\[
-W_{m,j} + W_{d,j} - \lambda_{d,j} = 0
\]

Using (4), it becomes

\[
\phi_j \lambda_{d,j} - \lambda_{d,j} = 0
\]  

(6)

3.2.2 Consumers

Consumers must choose the consumption quantities \( q_n \) and \( q_f \) to be purchased with each currency and the amount of loans in each currency, \( l_n \) and \( l_f \), before the second shock; i.e., before learning if they have a preference for the market good or for the home-made good. Their program is:

\[
\max_{q_j,l_j} \left[ \frac{b}{1-s} (u(q) + W (m_{b,n}, m_{b,f}, 0, 0, l_n, l_f)) + \frac{1-b-s}{1-s} W (m_{c,n}, m_{c,f}, 0, 0, l_n, l_f) \right] \\
\text{s.t. } p_j q_j \leq m_{-1,j} + l_j, \quad l_j \leq \bar{l}_j, \quad m_{b,j} = m_{-1,j} + l_j - p_j q_j, \quad m_{c,j} = m_{-1,j} + l_j, \quad q = q_n + q_f
\]

where \( m_{b,n} \) (\( m_{b,f} \)) and \( m_{c,n} \) (\( m_{c,f} \)) are the amounts of national (foreign) currency that a buyer and a home-consumer bring into the second market, respectively, and \( \bar{l}_j \) is the borrowing limit.

The first-order condition on \( q_j \) is:

\[
u'(q) - W_{m,j} p_j - \lambda_j p_j = 0
\]

where \( q = q_n + q_f \) and \( \lambda_j \) is the multiplier associated to the cash constraint. Using (4) and (5), this condition becomes:

\[
u'(q) = 1 + \lambda_j / \phi_j
\]  

(7)
The first-order condition on \( l_j \) is:

\[
b [u'(q) / p_j + W_{l,j}] + (1 - b - s) (W_{m,j} + W_{l,j}) - \lambda_{l,j} = 0
\]

where \( \lambda_{l,j} \) is the multiplier associated to the borrowing constraint. We rewrite the first-order condition on \( l_j \) as follows:

\[
u'(q) = 1 + (1 - s) (i_j + \lambda_{l,j} / \phi_j) / b
\]  

(8)

3.3 Banks

Banks must hold a proportion \( r \) in the form of deposits for each unit of money loaned. Therefore, they solve the following problem per borrower:

\[
\max_{l_j} \sum \phi_j l_j (i_j - r \bar{i}_{d,j} - \kappa)
\]

s.t. \( l_j \leq \bar{l}_j, \ b [u(q) - \phi_j (1 + i_j) l_j] - (1 - b - s) \phi_j i_j l_j \geq \Gamma_j
\]

where \( \kappa \) is the bank's operating cost per unit of money loaned and \( \bar{l}_j \) is the borrowing limit in each currency to be endogenized later. The first constraint is the borrowing constraint. The second constraint is the participation constraint of the borrower: each bank has to offer a pay-off to the borrower that is at least the same as the pay-off he may get while resorting to another bank, \( \Gamma_j \).

The first-order condition on \( l_j \) is:

\[
\phi_j (i_j - r \bar{i}_{d,j} - \kappa) - \lambda_{l,j} + \lambda_{\Gamma,j} \left[ bu'(q) \frac{dq}{dl} - b \phi_j - (1 - s) \phi_j i_j \right] = 0
\]

where \( \lambda_{l,j} \) and \( \lambda_{\Gamma,j} \) are the multipliers associated to the borrowing constraint and the participation constraint, respectively. By using (5), it becomes

\[
\phi_j (i_j - r \bar{i}_{d,j} - \kappa) - \lambda_{l,j} + \lambda_{\Gamma,j} \phi_j [bu'(q) - b - (1 - s) i_j] = 0
\]

3.4 Marginal value of money

The expected utility for an agent who starts a period with amounts \( m_n \) and \( m_f \) of each currency is:

\[
V(m_n, m_f) = b [u(q) + W(m_{b,n}, m_{b,f}, 0, 0, l_n, l_f)] \\
+ s [-q^s + W(m_{s,n}, m_{s,f}, d_n, d_f, 0, 0)] \\
+ (1 - b - s) W(m_{c,n}, m_{c,f}, 0, 0, l_n, l_f)
\]
Using (4), (6) and (7), the marginal value of currency \(j\) is:

\[ V_{m,j} = \phi_j [bu'(q) + s\delta d j + 1 - b] \]

Using (1) and (3), we have:

\[ \gamma_j / \beta - 1 = b [u'(q) - 1] + s\delta d j \]  

(9)

The right-hand side of this equation represents the marginal cost of acquiring an additional unit of currency \(j\) while the left-hand side represents its marginal benefit given by the increase in consumption \(q\) with probability \(b\) and in interests earned with probability \(s\).

### 3.5 Market clearing

In a symmetric equilibrium, the market-clearing conditions for the first market and the credit market are:

\[ sq_j^s = bq_j \]  

(10)

\[ sd_j = r (1 - s) l_j \]

Total output \(Y\) in the second market is:

\[ Y = by_b + sy_s + (1 - b - s) y_c \]

where \(y_b\), \(y_s\) and \(y_c\) are the amount of hours worked in the second subperiod of each period by the buyer, the seller and the home-consumer.\(^7\) Replacing \(y_b\), \(y_s\) and \(y_c\) and simplifying,

\[ Y = x + \frac{(1 - s) sq}{s + r (1 - s) \kappa} \]

(11)

The output produced in the second market is partly consumed by agents and partly used to afford the operating cost by banks.

### 4 Debt enforcement technology

In this section, we study the three cases regarding debt enforcement: perfect enforcement, no enforcement and imperfect enforcement. We will mostly focus on national-currency equilibria; i.e., equilibria in which only national currency is used \((\phi_n > 0 \text{ and } \phi_f = 0)\). Hence, we will ignore the subscripts to indicate the currency whenever they are not necessary.\(^8\) In order

\(^7\)The calculation of hours worked for this and next section is presented in the appendix.

\(^8\)Furthermore, we will not consider equilibria in which agents do not distinguish between currencies, as if they were not physically distinguishable.
to analyze the properties of different equilibria, we use (10) and (11) to state the expected lifetime utility $W$ for the representative agent as:

$$W (1 - \beta) = bu (q) - \left[ b + \frac{s(1-s)\kappa}{s + r(1-s)} \right] q + U (x) - x$$ (12)

### 4.1 Perfect enforcement

First, we analyze the national-currency unconstrained-credit equilibrium; i.e. an equilibrium in which agents may borrow as much as they desire since banks have the power to fully enforce all agents, regardless of their entering the second market or not. (9) becomes:

$$n = 1 = b [u_0 (q) - 1] + s i_d$$ (13)

In this equilibrium, $\lambda_l = 0 \ (\bar{\lambda} = \infty)$. Hence, from (8):

$$u' (q) = 1 + (1 - s) i/b$$ (14)

Besides, banks are competitive so banks’ profits are zero; this implies:

$$r i_d = i - \kappa$$ (15)

We can now define the national-currency equilibrium with perfect enforcement:

**Definition 1** A national-currency equilibrium with perfect enforcement is $\{q, i, i_d\}$ that satisfy (13), (14) and (15).

**Proposition 1** For $\gamma_n \leq \gamma_f$, a national-currency equilibrium with credit when enforcement is perfect exists if:

$$\frac{\gamma_n}{\beta} - 1 \geq (1 - s) \kappa$$ (16)

$$\frac{\gamma_n}{\beta} - 1 \leq \kappa / (1 - r)$$ (17)

The higher $r$ and $s$ the more likely that (16) and (17) are verified. When $r$ is high it is more likely that $i_d < i$ and that the zero-profit condition is satisfied. On the other hand, when $s$ is high, agents are less likely to become borrowers and pay the cost $\kappa$: credit is more useful when $s$ is high because agents are less willing to hold money across periods if they are producers more frequently.

Assume that only the national currency is available. Then, from Definition 1, we verify that $di/dr \geq 0$ and $d i_d / dr \leq 0$. If $r$ increases, the borrower has to pay more for each real

---

9To simplify the notation, we ignore utility by home-consumers in the first market since this would not alter the results presented.
unit of money loaned, whereas $i_d$ decreases because banks must compensate more units of money per loan. The consumption quantity, the individual amount borrowed and the real price of money are decreasing in $r$ since $dq/dr < 0$, $dl/dr < 0$ and $d\phi/dr < 0$. The effect of $\gamma_n$ on the endogenous variables is summarized by the following derivatives: $di/d\gamma_n \geq 0$, $di_d/d\gamma_n > 0$, $dq/d\gamma_n < 0$ and $d\phi/d\gamma_n < 0$.

Having defined the unconstrained equilibrium, we aim to determine if its existence is conditioned on the absence of a less inflationary currency. We state the following proposition:

**Proposition 2** The national-currency equilibrium with perfect enforcement is eliminated after an increase in $\gamma_n$ at $\gamma_n = \gamma_f$.

According to Proposition 2, the unconstrained national-currency equilibrium is not robust to the availability of a less inflationary foreign currency (i.e., the ability of agents to go to a country where the less inflationary currency circulates). Agents may choose which currency to use by staying or moving to another country. Since they can get a higher utility (from a higher consumption quantity) by using the less inflationary currency, the national-currency equilibrium cannot be sustained for $\gamma_n > \gamma_f$.

Furthermore, for $\gamma_n > \gamma_f$, agents choose to hold only foreign currency. If both currencies were held across periods, from (9) two first-order conditions on money should be satisfied:

$$\gamma_n/\beta - 1 = b[u'(q) - 1] + si_d,n$$
$$\gamma_f/\beta - 1 = b[u'(q) - 1] + si_d,f$$

where $q = q_n + q_f$. Using (15) and (8) to replace $i_d,n$ and $i_d,f$, they become:

$$\gamma_n/\beta - 1 = b\{1 + s/ [r (1 - s)]\} [u'(q) - 1] - s\kappa/r$$
$$\gamma_f/\beta - 1 = b\{1 + s/ [r (1 - s)]\} [u'(q) - 1] - s\kappa/r$$

The left-hand side (LHS) of each of these equations represents the marginal cost of getting an additional unit of national (foreign) currency whereas the right-hand side (RHS) represents the marginal benefit of getting an additional unit of national (foreign) currency. If $\gamma_n \neq \gamma_f$, one of these two conditions cannot be satisfied since RHS is the same in both of them. The agent chooses the currency with the lowest marginal cost, in this case the foreign currency since $\gamma_n > \gamma_f$.

For the rest of the paper, we will only consider the case where $\gamma_n \geq \gamma_f > \beta$ since our purpose is to determine conditions under which a domestic more-inflationary currency is used in equilibrium.
4.2 No enforcement

Next, we assume that no enforcement is feasible; i.e., banks are not able to enforce agents, regardless of agents entering the centralized market or not. Hence, banks have to set a borrowing constraint so that all agents (agents who prefer to consume their home-made good and agents who prefer the market good) choose to repay loans instead of defaulting on them.

The first-order condition on money is like in (13):

$$\frac{\gamma_n}{\beta} - 1 = b [u'(q) - 1] + si_d$$ \hspace{1cm} (18)

with

$$ri_d = i - \kappa \hspace{1cm} (19)$$

After defaulting, it is easy to verify that agents will use the foreign currency because they cannot use the banking system and national-currency inflation is assumed to be equal or higher than foreign-currency inflation. Thus, an agent who defaults in $t$ will move to the country where foreign currency is used during the settlement stage in $t$ and will use the foreign currency from then on. The marginal value of money for a defaulter satisfies:

$$\frac{\gamma_f}{\beta} - 1 = b [u'(\hat{q}) - 1] \hspace{1cm} (20)$$

where $\hat{q}$ is the quantity consumed by a defaulter. We denote $\hat{m}_{-1,f}$ the holdings of money by a defaulter at the end of the period. Hence, we have:

$$\hat{m}_{-1,f} = pf\hat{q}$$

$$\hat{m}_{+1,f} = \gamma_f\hat{m}_{-1,f} \hspace{1cm} (21)$$

In a symmetric equilibrium, money holdings by a non-defaulter are $m_{-1} = M_{-1}$. Since $\gamma_n > \beta$, the deposit constraint and cash constraint bind, so that money holdings by a non-defaulter satisfy:

$$m_{-1} = d = r(1 - s)l/s$$

$$pq = m_{-1} + l \hspace{1cm} (22)$$

Using (5) and (22) we can write the amount of an individual loan in real terms as:

$$\phi l = \frac{sq}{s + r(1 - s)} \hspace{1cm} (23)$$

Let us analyze next the decision by a buyer on whether to repay his loan or not before entering the second market in the period in which he borrowed. We call $W_b$ the expected lifetime utility if he does not default and $\tilde{W}_b$ his expected lifetime utility if he defaults:

$$W_b = U(x) - y_b + \beta V_{+1}$$

$$\tilde{W}_b = U(x) - \tilde{y}_b + \beta \tilde{V}_{+1}$$
where $y_b$ and $\bar{y}_b$ are the amounts of hours worked by the buyer who does not default and the buyer who defaults in the current period, respectively. $V + 1$ corresponds to the expected lifetime value for a buyer who defaults in the current period.

Banks set a borrowing constraint in order to prevent default. They choose $\bar{l}$, the maximum amount loaned, such that the expected lifetime utility for a non-defaulter equals the expected lifetime utility for a defaulter. In the case of the buyer, the borrowing constraint may be written as follows:

$$-y_b + \frac{\beta}{1 - \beta} \left[ bu(q) - sq^* + U(x) - by_b - s\hat{y}_s - (1 - b - s)y_c \right]$$

where $y_b$, $\hat{y}_s$, and $y_c$ are the amounts of hours worked in the second subperiod of each period by the buyer, the seller or the home-consumer who are not defaulters and did not default in the past, respectively. $\bar{y}_b$, $\hat{y}_s$, and $\hat{y}_c$ are the hours worked by a defaulter the period after defaulting and from then on, and $\hat{q}^*$ is the amount produced in the first market by the defaulter each time he turns out to be a seller once he moved to the economy where the foreign currency is used. If an agent defaults, he saves working hours since he does not repay the loan nor the interest (i.e.; $\bar{y}_b < y_b$), but he is punished by being permanently excluded from the banking system, so that his consumption and working hours as a defaulter may be different from those as non-defaulter.

Since the consumer’s decision on defaulting may depend on the consumer’s type, we should also consider the decision by the home-consumer. We state the following Lemma:

**Lemma 1** The pay-off to a defaulter is independent of his type (buyer or home-consumer). The borrowing constraint set by banks when no enforcement is feasible is

$$\bar{\phi} \left[ i + 1 + \beta (1 - s) \kappa / (1 - \beta) \right]$$

$$= \beta b \left[ u(q) - u(\hat{q}) - (q - \hat{q}) \right] / (1 - \beta) + (\gamma_f - \beta) \hat{q} / (1 - \beta) - \gamma_n (q - \bar{\phi} \bar{l})$$

The left-hand side in (25) is the gain from defaulting and the right-hand side is the cost of defaulting. The defaulter avoids working to pay back both the loan and the interest in the period in which he defaults ($\bar{\phi} \bar{l} (i + 1)$). In addition, since he will be excluded from the banking system for the rest of his lifetime, he will not face the expected cost of having access to it ($\bar{\phi} \bar{l} \beta (1 - s) \kappa / (1 - \beta)$). The opportunity cost of defaulting is given by the difference in the net utility of consuming $\hat{q}$ instead of $q$ each time the agent turns out to be buyer ($\beta b \left[ u(q) - u(\hat{q}) - (q - \hat{q}) \right] / (1 - \beta)$). In addition, since the defaulter cannot resort to banks he needs to acquire extra money holdings each period (($\gamma_f - \beta) \hat{q} / (1 - \beta)$). Since he
uses foreign currency after defaulting, he spends his domestic money holdings in the period of default \((-\gamma_n (q-\phi l))\).

According to Lemma 1, the pay-off to a defaulter is the same regardless of his being a buyer or a home-consumer. Indeed, the expected lifetime utility from the period after default is the same. The gain from defaulting in the period of default is also the same because they both save \(\phi l (1+i)\) working hours: At the moment of defaulting, the buyer has already spent the money borrowed to buy goods in the first market so he avoids working to buy \(l (1+i)\) units of money, while the home-consumer does not need to work to buy \(il\) units of money and uses the money borrowed \(l\) to get his optimal money holdings working less than otherwise.

When the borrowing constraint binds, \(l = \bar{l}\). To rewrite (25), we set \(l = \bar{l}\) and make use of (23):

\[
\frac{sq}{(1-s) r + s} = \frac{\beta b [u(q) - u(q)] - [\gamma_f - \beta (1-b)] q + \gamma_n [\beta b + (1-\beta) \gamma_n] q}{(\gamma_n - 1 - \bar{l}) (1-\beta) - \beta (1-s) \kappa} 
\]

(26)

We can now state the following definition:

**Definition 2** A national-currency constrained-credit equilibrium with no enforcement is \(\{q, \hat{q}, i, i_d\}\) that satisfy (18), (19), (20) and (26).

We call \(\bar{\gamma}\) the value of \(\gamma_n\) such that \(i_d = 0\). We write (26) setting \(i_d = 0\) (or \(i = \kappa\)) and \(\gamma_n = \gamma_f = \bar{\gamma}\),

\[
\frac{sq}{(1-s) r + s} = \frac{\beta b [u(q) - u(q)] + (\beta b + \bar{\gamma}) (q - \hat{q}) + \beta (\hat{q} - \bar{\gamma} q)}{(\bar{\gamma} - 1) (1-\beta) - \kappa (1-\beta s)}
\]

We know that \(q = \hat{q}\) when \(\gamma_n = \gamma_f = \bar{\gamma}\); thus,

\[
\bar{\gamma} = 1 + \frac{s (1-\beta s) \kappa}{s + (1-s) r \beta} 
\]

(27)

Hence, as long as \(\kappa > 0\), \(\bar{\gamma} > 1\) (if \(\kappa = 0\), \(\bar{\gamma} = 1\)). To see why, consider the case in which \(\kappa = 0\). The gain for an agent who defaults consists of the working hours saved in the current period, \((1+i) \phi l = \phi l\). The cost of defaulting is given by the working time in each period necessary to acquire extra money holdings, which is equal to \(\phi l_{t+1} - \beta \phi l_{t+1} = (\gamma_n - \beta) \phi l\). The discounted lifetime sum of this cost is \(\sum_{t=0}^{\infty} \beta^t (\gamma_n - \beta) \phi l = (\gamma_n - \beta) \phi l / (1-\beta)\). Hence, \(\bar{\gamma} = \gamma_n = 1\) corresponds to the level of \(\bar{\gamma}\) for which the agent is just indifferent between defaulting or not when \(\kappa = 0\). As a consequence, when \(\kappa > 0\), \(\bar{\gamma}\) must be higher than one; otherwise the gain for a defaulter would be higher than in the case with \(\kappa = 0\) but the

\(^{10}\)When the money growth rate is equal to 1 there are no lump-sum monetary transfers, so we do not need to consider them in the cost of defaulting in this case.
cost would be the same, which cannot occur in equilibrium. Hence, when credit is costly the inflation rate must be positive for an equilibrium with credit to exist.

At $\gamma_n = \bar{\gamma}$, from (18) $u'(q)$ is:

$$u'(q) = \frac{\gamma_n - \beta}{b\beta} + 1$$

(28)

We now differentiate (26) with respect to $\gamma_n$ (after replacing $i$ with (18) and (19)):

$$\frac{dq}{d\gamma_n} = \frac{(1 - s + 1/\beta)(1 - \beta)rq}{(\gamma_n - 1)[s + \beta(1 - s)r] - s(1 - \beta s) \kappa + (1 - \beta)rbqu''(q)}$$

Notice that we have set $dq/d\gamma_n = 0$ according to (20) and used (28) to replace $u'(q)$. Replacing $\gamma_n = \bar{\gamma}$ with (27), we verify that $dq/d\gamma_n$ evaluated at $\bar{\gamma}$ is negative:

$$dq/d\gamma_n|_{\bar{\gamma}} = (1 - s + 1/\beta) / [bu''(q)]$$

(29)

Therefore, when no enforcement is feasible, increasing the domestic money growth $\gamma_n$ reduces the quantity consumed by a non-defaulter who uses the domestic currency.

**Proposition 3** The constrained-credit national-currency equilibrium with no enforcement is eliminated after an increase in $\gamma_n$ at $\bar{\gamma} = \gamma_n = \gamma_f$.

The explanation for Proposition 3 relies on the usual effect caused by an increase in the inflation rate: If only one currency were available, agents would choose to hold a lower amount of money when inflation increases and consume less so that the marginal value of money increases as well. The deposit interest rate is higher for a higher inflation rate but not sufficiently higher so as to compensate for the higher cost of holding national currency. The reason is that an increase in $\gamma_n$ does not modify the outside option for defaulters, which depends only on $\gamma_f$. Therefore, the borrowing constraint for loans in national currency is not relaxed after an increase in $\gamma_n$, which precludes an increase in the deposit interest rate sufficient to make agents hold the national currency at the end of the period. Since a second currency is available, agents would switch to the foreign currency if $\gamma_n$ increases and $\gamma_f$ does not, in order to enjoy a higher consumption. Hence, the national-currency equilibrium cannot be supported for $\gamma_n > \gamma_f$ when banks have no enforcement power.

### 4.3 Imperfect enforcement

Finally, we consider the case of imperfect enforcement; i.e., banks are able to enforce agents who voluntarily enter the second market and trade. However, they cannot enforce agents who do not take part in the second market. Agents who prefer the market good will always
enter the market, since they would not be able to consume otherwise and doing so would entail infinite negative utility (\( \ln(0) = -\infty \)). Therefore, banks are concerned only with home-consumers who do not need to enter the market to consume. Indeed, home-consumers could decide not to enter the market in order to avoid being enforced and save working effort. Banks set a borrowing constraint to prevent default by these agents. They choose \( \bar{l} \) such that the expected lifetime utility for a home-consumer who does not default equals the expected lifetime utility for a home-consumer who defaults; i.e. \( W_c = \bar{W}_c \), with:

\[
\begin{align*}
W_c &= U(x) - y_c + \beta V_{+1} \\
\bar{W}_c &= U(x) - y^*_c + \beta \bar{V}_{+1}
\end{align*}
\]

where \( y^*_c \) is the amount of hours worked by the defaulter in the period he defaults and \( \bar{V}_{+1} \) corresponds to the expected lifetime value for a defaulter in that period. A defaulter will move to the country where the foreign currency is used during the settlement stage of the period after defaulting, since he cannot move while banks can enforce him. The borrowing constraint \( W_c = \bar{W}_c \) is then:

\[
\begin{align*}
-y_c + \frac{\beta}{1-\beta} [bu(q) - sq^* + U(x) - by_b - sy_s - (1-b-s) y_c] \\
= -y^*_c + \beta [bu(\bar{q}) - sq^* + U(x) - b\bar{y}_b - s\bar{y}_s - (1-b-s) \bar{y}_c] \\
+ \frac{\beta^2}{1-\beta} [bu(\bar{q}) - s\bar{q}^* + U(x) - b\bar{y}_b - s\bar{y}_s - (1-b-s) \bar{y}_c]
\end{align*}
\]

where \( \bar{y}_b, \bar{y}_s \) and \( \bar{y}_c \) are the amounts of hours worked by the defaulter the period after he defaults if he turns out to be buyer, seller or home-consumer, respectively; \( \bar{y}_b, \bar{y}_s \) and \( \bar{y}_c \) are the hours worked by a defaulter the second period after defaulting and from then on, and \( \bar{q}^* \) is the amount produced in the first market by the defaulter each time he turns out to be a seller once he moved to the economy where the foreign currency is used. \( \bar{q} \) is the quantity consumed by the defaulter the period after defaulting if he happens to be buyer.

In the period after defaulting the agent holds the money borrowed and not reimbursed. If the agent is home-consumer or seller in that period, he uses this money to save working hours in the second market. If he is buyer in this period, he uses this money to consume in the first market so that working hours in the period after defaulting are the same as in all other periods. So we verify that \( \bar{y}_b = \bar{y}_b \) (see appendix).

The quantity consumed by the defaulter if he is buyer the period after defaulting is:

\[
\bar{q} = (m_{-1} + l_{-1}) / p
\]

since his money holdings when entering the first market are the sum of his previous money holdings and the defaulted loan from the previous period.\(^{11}\) Since \( l \) is a multiple of \( m_{-1} \) (\( l_{-1} \)

\(^{11}\)Note that the defaulter receives monetary transfers the same period of default, even though he will not receive them from then on, since the default is noticed only at the end of the period.
is a multiple of \( m_{-2} \) and \( m \) grows at a rate \( \gamma_n \), it follows that \( l_{-1} = l / \gamma_n \). Using (5), (22) and (23) we can rewrite (31) as:

\[
\hat{q} = q \frac{s / \gamma_n + r (1 - s)}{s + r (1 - s)}
\]

Thus, \( \hat{q} = q \) if \( \gamma_n = 1 \) and \( \hat{q} < q \) if \( \gamma_n > 1 \).

**Lemma 2** The borrowing constraint set by banks when imperfect enforcement is feasible is

\[
\phi \hat{l} [i + \beta (1 - b) / \gamma_n + \beta \kappa (1 - s) / (1 - \beta)] / \beta
\]

\[
= b [u(q) - \phi \hat{l} - u(\hat{q})] + \beta b [u(q) - u(\hat{q}) - (q - \hat{q})] + (\gamma_f - \beta) \hat{q} / (1 - \beta) + \phi \hat{l} - q
\]

The borrowing constraint in (33) has a similar interpretation as in (25). The difference is that the pay-off in the first period after defaulting is different from the pay-off to a defaulter during the rest of his lifetime, because just after defaulting he holds the money not reimbursed and he has not been able yet to switch currencies or adjust his money holdings. The left-hand side in (33) is different from the one in (25) in the term \( \beta \phi \hat{l} (1 - b) / \gamma_n \): When enforcement is imperfect, the gain from defaulting is given by the real value of the defaulted loan in the period after default, which depends on \( \gamma_n \). If in this period the agent is a seller or a home-consumer, he can spend the loan in the second market. If the agent is a buyer in that period, he gets utility \( u(\hat{q}) \), instead of \( u(q) - \phi \hat{l} \).

Setting \( \phi \hat{l} = \phi \bar{l} \) in (33) and using (23) yields

\[
\frac{(1 - \beta) s q}{\beta [s + r (1 - s)]} = b [u(q) - (1 - \beta) u(\hat{q}) - \beta u(\hat{q})] + \beta (1 - b) (q - \hat{q}) + \gamma_f \hat{q} - q
\]

\[
= b [u(q) - (1 - \beta) u(\hat{q}) - \beta u(\hat{q})] + \beta (1 - b) (q - \hat{q}) + \gamma_f \hat{q} - q
\]

\[
= b [u(q) - (1 - \beta) u(\hat{q}) - \beta u(\hat{q})] + \beta (1 - b) (q - \hat{q}) + \gamma_f \hat{q} - q
\]

We can now state the following definition:

**Definition 3** A national-currency constrained-credit equilibrium with imperfect enforcement is \( \{q, \hat{q}, \bar{q}, i, i_d\} \) that satisfy (18), (19), (20), (32) and (34).

We call \( \bar{q} \) the value of \( \gamma_n \) such that \( i_d = 0 \) when enforcement is imperfect. If \( \gamma_n = \gamma_f = \bar{q} \), from (18) and (20) \( q = \hat{q} \); thus,

\[
\frac{\kappa s (1 - \beta s) / [\beta (1 - \beta)]}{s + r (1 - s)} = \frac{b [u(q) - u(\hat{q})]}{q} + \frac{\bar{q} - 1}{1 - \beta} - \frac{s (1 - b) (1/\bar{q} - 1)}{s + r (1 - s)}
\]

The solution for \( \bar{q} \) is implicitly determined by (35), where \( q \) and \( \hat{q} \) are functions of \( \bar{q} \) and are determined by (18) and (32) when \( \gamma_n = \bar{q} \) and \( i_d = 0 \).
Lemma 3 For $\kappa > 0$, the money growth rate consistent with zero nominal interest rate when imperfect enforcement is feasible, $\tilde{\gamma}$, is higher than 1.

Lemma 3 implies that the inflation rate must be positive to support an equilibrium with credit, as in the case with no enforcement. Before evaluating how the borrowing constraint is affected by an increase in $\gamma_n$, we calculate $d\tilde{q}/d\gamma_n$ from (32):

$$
d\tilde{q}/d\gamma_n [s + r (1 - s)] = (dq/d\gamma_n) [s/\gamma_n + r (1 - s)] - sq/ (\gamma_n)^2
$$

The effect of $\gamma_n$ on the consumption by the defaulter in the period after defaulting if he turns out to be a buyer consists of two terms. The second term in the right-hand side of (36) corresponds to the depreciation of the loan owing to an increase in $\gamma_n$ and it is always negative. The first term corresponds to the variation in the price of money $\phi$ when $\gamma_n$ increases, which is reflected in the change in $q$. If $q$ increases after an increase in $\gamma_n$, this means that $\phi$ increases which allows for a higher $\tilde{q}$.

Differentiating (34) with respect to $\gamma_n$ (after replacing $i$ with (18) and (19)) and evaluating at $\gamma_n = \gamma_f = \tilde{\gamma}$ yields

$$
dq/d\gamma_n|_{\tilde{\gamma}} = \frac{qr/\beta - q\beta s [bu'(\tilde{q}) - b + 1] / (\tilde{\gamma})^2}{\beta [s + r (1 - s)] \left\{ \frac{bu'(q) - \gamma + \beta (1 - b)}{1 - \beta} - \frac{b[u(q) - u(\tilde{q})]}{q} \right\} - \beta b [r (1 - s) + s/\tilde{\gamma}] u'(\tilde{q}) + bu''(q) q
$$

where we have used (36) and $d\tilde{q}/d\gamma_n = 0$.

Proposition 4 If $r \leq s\beta/\tilde{\gamma}$, the constrained-credit national-currency equilibrium with imperfect enforcement is not eliminated after an increase in $\gamma_n$ at $\gamma_n = \gamma_f = \tilde{\gamma}$.

If the solution for $\tilde{\gamma}$ from (35) satisfies $r \leq s\beta/\tilde{\gamma}$, then $dq/d\gamma_n > 0$ at $\gamma_n = \gamma_f = \tilde{\gamma}$. Hence, $r \leq s\beta/\tilde{\gamma}$ is a sufficient condition for the national-currency equilibrium not to be eliminated after an increase in $\gamma_n$ when $\gamma_n = \gamma_f = \tilde{\gamma}$ and $\gamma_f$ is held fixed. Since $r \leq s\beta/\tilde{\gamma}$ ensures $dq/d\gamma_n > 0$, we also state the following proposition:

Proposition 5 If $r \leq s\beta/\tilde{\gamma}$ and enforcement is imperfect, after an increase in $\gamma_n$ at $\gamma_n = \gamma_f = \tilde{\gamma}$ welfare is higher in the constrained-credit national-currency equilibrium than in the foreign-currency constrained-credit equilibrium.
Thus, in the case of imperfect enforcement, under certain conditions, increasing $\gamma_n$ makes more profitable to hold the domestic currency. This implies that a constrained-credit national-currency equilibrium is robust to increasing $\gamma_n$ even if a less inflationary currency is available and no transaction costs specific to the use of foreign currency are assumed.

The explanation for this result resides in the link between the inflation rate of a currency and the borrowing constraint associated to that currency. Since defaulters are obliged to skip one period to use the defaulted loan, the inflation of the currency in which the loan is denominated matters. The more inflationary a currency is, the less valuable a defaulted loan denominated in that currency is when it may be used to purchase goods, and the smaller the incentives to default are. Consequently, the borrowing constraint on the loans denominated in a particular currency may be relaxed when the inflation rate of that currency increases. This, in turn, allows for an increase in the borrowing interest rates: Since the gain from defaulting is reduced with inflation, the cost of repayment may be increased without inciting default. The increase in the borrowing interest rate is reflected in the deposit interest rate to satisfy the zero-profit condition for banks. As a result, agents may prefer to take the more inflationary currency across periods: If they were to deposit their money holdings, they would be better compensated for deposits in the more inflationary currency than for deposits in the less inflationary currency. The higher demand for the former currency makes its price increase as well, which allows buyers to get a higher quantity of goods in the market, as illustrated by the derivative in (37) when $r \leq s\beta/\gamma$ holds.

The key feature of a constrained-credit equilibrium is that the increase in the deposit interest rate after an increase in the inflation rate may exceed the one that occurs in an unconstrained equilibrium. In general, when inflation increases, the deposit interest rates should increase to compensate agents for the highest cost of holding money. However, in a constrained equilibrium interest rates are set below their market-clearing level to prevent default. Hence, the relaxation of the borrowing constraint is an additional channel through which the interest rate increases after an increase in inflation. This additional channel is what explains the preference for a currency which loses purchasing power more rapidly than another.

The preference for the more inflationary currency is not possible when there is no enforcement, however, because the link between the inflation of a currency and the incentives to default loans denominated in that currency does not exist: Agents do not need to wait to use the money borrowed and so they spend it in the period when they default. Inflation in general does not affect their outside option, which is entirely determined by the currency that they will use after defaulting. If the outside option consists in using the foreign currency, increasing domestic inflation has no effect on it. The borrowing constraint is also unaffected and the mechanism through the interest rates described in the previous paragraph cannot
take place. Therefore, an increase in the domestic inflation makes agents switch to the foreign currency in this case.

Let us now explain the condition stated in Propositions 4 and 5, by which the level of reserve requirements must be sufficiently low for inflation to increase the quantity consumed by the non-defaulter. This condition is obtained from a simplification of the terms in the numerator in (37). The term $r/\beta$ comes from the increase in the borrowing interest rate when $\gamma$ increases and is lower the lower is $r$. The term $s/\gamma$ comes from the decrease in the real value of the defaulted loan when it may be spent to purchase goods, owing to an increase in $\gamma_n$ at $\gamma_n = \gamma$. Asking $r$ to be low enough is equivalent, then, to ask that the depreciation of the money borrowed is more important than the increase in the interest rate when $\gamma_n$ increases, so that the borrower is less willing to default.

4.3.1 Numerical example

To get an example where the more inflationary currency is preferred, we assume $u(q) = q^n$, with $n < 1$. We give arbitrary values to all parameters except $\gamma_n$ and $\gamma_f$. We set $r = 0.2$, $b = 0.4$, $s = 0.48$, $\eta = 0.8$, $\kappa = 0.02$ and $\beta = 0.9$. We also set $\gamma_n = \gamma_f = \gamma$ and compute $\gamma$ using (35), where the solution for $\bar{q}$ comes from (32) and the solution for $q$ comes from (18) when $\gamma_n = \gamma$ and $i_d = 0$. We get $\gamma = 1.00951$. We verify that $r \leq s\beta/\gamma$ since $s\beta/\gamma = 0.428 > r = 0.2$. We also verify that (16) holds, since $(\gamma - \beta)/\beta = 0.1217 > \kappa (1 - s) = 0.01$, which ensures existence of credit. This means that $q$ is increasing in $\gamma_n$ at $\gamma_n = \gamma_f = \gamma$. Then, we can compute $q$ from (34), where $\bar{q}$ and $\check{q}$ are defined by (20) and (32), respectively.

In Figure 2 we have plotted $q$ as a function of $\gamma_n$, from $\gamma_n = \gamma_f = \gamma$ to the value of $\gamma_n$ for which the wedge between the borrowing interest rate and the deposit interest rate is zero (for higher values of $\gamma_n$, the wedge becomes negative which precludes the existence of a national-currency equilibrium with credit).

![Figure 2: Consumption by the non-defaulter as a function of $\gamma_n$](image-url)
Notice that, in this example, condition (16) holds but condition (17) does not. To ensure the existence of an unconstrained-credit equilibrium, both conditions were required. The first one states that domestic inflation must not be too low compared to the bank cost for agents to be willing to use credit instead of relying only on their money holdings. This condition applies for the definition of both an unconstrained-credit equilibrium and a constrained-credit equilibrium, since in both cases we require that agents choose to use the banking system instead of only using money.

\[ g_n \]

The second condition states that domestic inflation must not be too high for the banks’ zero-profit condition to be verified. If inflation is very high, the deposit interest rate is also high to compensate agents for the cost of carrying money. In addition, when inflation increases, the deposit interest rate increases faster than the borrowing interest rate owing to the zero profit condition and the existence of reserve requirements lower than 100%. Eventually, for \( g_n \) sufficiently high as defined in (17), the deposit interest rate would have to be higher than the borrowing interest rate to satisfy the zero profit condition, which cannot occur in equilibrium. However, this condition does not need to hold in the case of a constrained-credit equilibrium. In this case, the deposit interest rate is not at its market-clearing level: Since enforcement is not perfect, the borrowing interest rate is lower than its market-clearing level to prevent default by borrowers and so depositors cannot be fully remunerated. Hence, \( g_n \) may violate (17) in a constrained-credit equilibrium without implying that \( i_d > i \), as we can verify from Figure 3.

\[ i_d > i \]

\[ \gamma_n \]

Figure 3: Interest rates as a function of \( \gamma_n \).

5 Conclusion

We have presented a model where agents can choose between two currencies with potentially different inflation rates in order to identify conditions under which they prefer to hold and to
accept in trade the more inflationary currency, even though no specific costs or legal restrictions are associated to the less inflationary currency. We have shown that the borrowers' enforcement technology and the weight of loans relative to the money stock (i.e., the level of banks' reserve requirements) are important for the existence of national-currency equilibria when a foreign less inflationary currency is available.

Our set-up should be improved in a number of ways. On the one hand, the enforcement technology could be modeled as a policy variable. This would allow us to explicitly study combinations of the money growth rate and the policy variable for the choice of the level of enforcement. In addition, an important step would be to introduce a second country with potentially different characteristics. This would make possible to ask about the conditions for different currencies being used in different countries, among other questions, and to compare with a large body of literature that studies two-country set-ups. All this is part of future research.

References


Appendix

Hours worked in the case of no enforcement in the national-currency equilibrium

The amount of hours worked by the agent who does not default is:

\[ y_b = x + \phi (1 + i) l + \phi m_{+1} - \phi (\gamma_n - 1) m_{-1} = x + i\phi l + q \]
\[ y_s = x - \phi (1 + i_d) d - \phi pq^* + \phi m_{+1} - \phi (\gamma_n - 1) m_{-1} = x - i_d r (1 - s) \phi l / s - bq / s \]
\[ y_c = x + \phi (1 + i) l - \phi l + \phi m_{+1} - \phi m_{-1} - \phi (\gamma_n - 1) m_{-1} = x + \phi i l \]

The amounts of hours worked by the buyer and the home-consumer who default in the period of default and thereafter are:

\[ \bar{y}_b = x + \phi_f \hat{m}_{f+1} - \phi (\gamma_n - 1) m_{-1} = x + \gamma_f \hat{q} - (\gamma_n - 1) (q - \phi l) \]
\[ \bar{y}_c = x - \phi l + \phi_f \hat{m}_{f+1} - \phi m_{-1} - \phi (\gamma_n - 1) m_{-1} = x + (\gamma_n - 1) \phi l + \gamma_f \hat{q} - \gamma_n q \]
\[ \bar{y}_b = x + \phi_f \hat{m}_{f+1} = x + \gamma_f \hat{q} \]
\[ \bar{y}_s = x - \phi_f p_f^* \hat{q} - \phi_f \hat{m}_{f-1} + \phi_f \hat{m}_{f+1} = x - \hat{q}^* + (\gamma_f - 1) \hat{q} \]
\[ \bar{y}_c = x - \phi_f \hat{m}_{f-1} + \phi_f \hat{m}_{f+1} = x + (\gamma_f - 1) \hat{q} \]

Hours worked in the case of imperfect enforcement in the national-currency equilibrium

The amounts of hours worked by the non-defaulter are as in the case with no enforcement. The hours worked by the home-consumer when enforcement is imperfect in the period of default and the period after default are:

\[ y_c^* = x \]
\[ \bar{y}_b = x + \phi_f \hat{m}_{f+1} = x + \gamma_f \hat{q} \]
\[ \bar{y}_s = x + \phi_f \hat{m}_{f+1} - \phi m_{-1} - \phi l_{-1} - \phi pq^* = x + \gamma_f \hat{q} - (1 + b / s) q + (1 - 1 / \gamma_n) \phi l \]
\[ \bar{y}_c = x + \phi_f \hat{m}_{f+1} - \phi m_{-1} - \phi l_{-1} = x + \gamma_f \hat{q} - q + (1 - 1 / \gamma_n) \phi l \]

The hours worked by the defaulter in subsequent periods are the same as in the case with no enforcement.

Proof of Proposition 1

For an unconstrained-credit equilibrium to exist, it must be that \( i_d \geq 0 \) (or \( i \geq \kappa \)) and \( i \geq i_d \). Using (14) and (15) these conditions reduce to (16) and (17).
Proof of Proposition 2

Denote $Q_n$ the consumption of a buyer in an economy where only the national currency is used and $Q_f$ the consumption of a buyer in an economy where only the foreign currency is used. From (9), the first-order conditions on money are respectively $\gamma_n/\beta - 1 = b [u' (Q_n) - 1] + si_{d,n}$ and $\gamma_f/\beta - 1 = b [u' (Q_f) - 1] + si_{d,f}$. Using (15) and (8) to replace $i_{d,n}$ and $i_{d,f}$, they become:

$$\gamma_n/\beta - 1 = b \{1 + s/ [r (1 - s)]\} [u' (Q_n) - 1] - s\kappa / r$$
$$\gamma_f/\beta - 1 = b \{1 + s/ [r (1 - s)]\} [u' (Q_f) - 1] - s\kappa / r$$

Hence, for $\gamma_n > \gamma_f, Q_f > Q_n$. According to (12), a higher consumption quantity implies a higher $W$ since $b [u' (Q_j) - 1] - s (1 - s) \kappa / [s + r (1 - s)] = (\gamma_j/\beta - 1) r (1 - s) / [s + r (1 - s)] > 0$.

Proof of Lemma 1

The borrowing constraint (25) results from replacing $y_b, y_s, y_c, \bar{y}_b, \bar{y}_s, \bar{y}_c$ and $\bar{y}_c$ (for which (21), (22), (5) and (19) are taken into account) and setting $l = \bar{l}$ in (24). It is straightforward to verify that the same borrowing constraint results from equating the expected pay-offs from defaulting and not defaulting that correspond to a home-consumer.

Proof of Proposition 3

Denote $Q_n$ the consumption of a buyer in an economy where only the national currency is used and $Q_f$ the consumption of a buyer in an economy where only the foreign currency is used. At $\gamma_n = \gamma_f = \bar{\gamma}, Q_n = Q_f = \bar{Q}$. After an increase in $\gamma_n$, $Q_n < \bar{Q}$, given $dq/d\gamma_n|_{\bar{\gamma}} < 0$, whereas $Q_f = \bar{Q}$ since $\gamma_f$ is kept unchanged. According to (12), a lower consumption quantity implies a lower $W$ because $b [u' (Q_j) - 1] - s (1 - s) \kappa / [s + r (1 - s)] = \gamma_j/\beta - 1 - s (1 - s) \kappa / [s + r (1 - s)] > 0$ at $\gamma_j = \bar{\gamma}$ given that $\gamma_j/\beta - 1 \geq (1 - s) \kappa$ for an equilibrium with credit to exist.

Proof of Lemma 2

(33) comes from replacing $y_b, y_s, y_c, \bar{y}_b, \bar{y}_s, \bar{y}_c, \bar{y}_b$ and $\bar{y}_c$ in (30) and setting $l$ equal to $\bar{l}$ and $l_{-1}$ to $l/\gamma_n$.

Proof of Lemma 3

To verify that $\bar{\gamma} > 1$, we write (34) setting $i_d = 0$ and $\gamma_n = \gamma_f = \bar{\gamma}$,

$$\frac{(1 - \beta) q}{\beta \{s + r (1 - s)\}} = b \frac{[u (q) - (1 - \beta) u (\hat{q}) - \beta u (\hat{q})] + \beta (1 - b) (q - \hat{q}) + \bar{\gamma} \hat{q} - q}{s\beta (1 - b) (1/\bar{\gamma} - 1) + (1 - \beta \kappa s) / (1 - \beta)}$$
If \( \tilde{\gamma} = \gamma_n = \gamma_f, \ q = \tilde{q} \) from (18) and (20); thus,
\[
\frac{(1 - \beta s) \kappa s}{(1 - \beta) \beta} = [s + r (1 - s)] \left\{ b [u(q) - u(\tilde{q})]/q + \frac{\tilde{\gamma} - 1}{1 - \beta} \right\} + s (1 - b) (1 - 1/\tilde{\gamma}) \tag{38}
\]

If \( \tilde{\gamma} \) were 1, LHS would be higher than RHS in (38). We can verify that RHS increases with \( \gamma_n \) when setting \( \gamma_n \) to 1. The derivative of RHS with respect to \( \gamma_n \) is:
\[
(dRHS/d\gamma_n)/\beta = b \{(dq/d\gamma_n) [(u'(q) - u(q)/q + u(\tilde{q})/q)] - (d\tilde{q}/d\gamma_n) u'(\tilde{q})/q \}
+ s (1 - b) / \left\{ [s + r (1 - s)] (\gamma_n)^2 \right\} + 1/ (1 - \beta)
\]

Using the expression for \( d\tilde{q}/d\gamma_n \) in (36) to rewrite (39) and evaluating it at \( \tilde{\gamma} = \gamma_n = \gamma_f = 1 \) yields:
\[
(dRHS/d\gamma_n)|_{\tilde{\gamma}=1} = \frac{s \beta [1 + b (u'(q) - 1)]}{s + r (1 - s)} + \frac{\beta}{1 - \beta} > 0
\]

Therefore, \( \tilde{\gamma} \) should be higher than one to satisfy this constraint.

**Proof of Proposition 4 and Proposition 5**

First, we show that \( dq/d\gamma_n \rvert_{\tilde{\gamma}} > 0 \). The denominator in (37) may be rewritten as follows by using the mean value theorem:
\[
\frac{bu'(q) - \tilde{\gamma} + \beta (1 - b)}{1 - \beta} - \frac{b [r (1 - s) + s/\tilde{\gamma}] u'(\tilde{q}) - ru''(q) q/\beta}{s + r (1 - s)} - bu'(q_m) (1 - \tilde{q}/q)
\]

where \( q_m \in (\tilde{q}, q) \) is such that \( u'(q_m) (q - \tilde{q}) = u(q) - u(\tilde{q}) \). We set \( u'(\tilde{q}) = u'(q) + \epsilon \) and \( u'(q_m) = u'(q) + \varepsilon \), with \( \epsilon, \varepsilon > 0 \). Using (32) and rearranging:
\[
\frac{\beta bu'(q) - \tilde{\gamma} + \beta (1 - b)}{1 - \beta} + \frac{br u''(q) q/\beta - \epsilon [r (1 - s) + s/\tilde{\gamma}] - \varepsilon (s - s/\tilde{\gamma})}{s + r (1 - s)}
\]

Finally, we replace \( u'(q) \) using (20) and simplify so that all remaining terms are negative:
\[
uu''(q) q/\beta - \epsilon [r (1 - s) + s/\tilde{\gamma}] - \varepsilon (s - s/\tilde{\gamma}) < 0
\]

For the numerator in (37) to be also negative it must be that:
\[
r/\beta^2 - s \{1 - b + [u'(q) + \epsilon] b\} / (\tilde{\gamma})^2 < 0
\]

We use (20) to replace \( u'(q) \) (which is equal to \( u'(\tilde{q}) \) for the parameters values that we are examining) and simplify:
\[
(r/\beta - s/\tilde{\gamma})/\beta < sb\epsilon / (\tilde{\gamma})^2
\]

We write it as a sufficient condition by ignoring the RHS and rearrange to get to \( r \leq s\beta/\tilde{\gamma} \), which implies \( dq/d\gamma_n \rvert_{\tilde{\gamma}} > 0 \). Denote \( Q_n \) the consumption of a buyer in an economy where only the national currency is used and \( Q_f \) the consumption of a buyer in an economy
where only the foreign currency is used. At $\gamma_n = \gamma_f = \tilde{\gamma}$, $Q_n = Q_f = \tilde{Q}$. After an increase in $\gamma_n$, $Q_n > \tilde{Q}$, given $dq/d\gamma_n|_q > 0$, whereas $Q_f = \tilde{Q}$ since $\gamma_f$ is kept unchanged. According to (12), a higher consumption quantity implies a higher $W$ because $b[u'(Q_j) - 1] - s(1 - s)\kappa/s + r(1 - s)] = \gamma_j/\beta - 1 - s(1 - s)\kappa/[s + r(1 - s)] > 0$ at $\gamma_n = \gamma_f = \tilde{\gamma}$ given that $\gamma_j/\beta - 1 \geq (1 - s)\kappa$ for an equilibrium with credit to exist.