Upstream capacity constraint and the preservation of monopoly power in private bilateral contracting

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Abstract

This article presents a model of private vertical contracting with a capacity constrained monopolistic supplier. I consider ‘full capacity beliefs’ that are consistent with an upstream capacity constraint and are ‘wary’ when the constraint is tight or production is costless. I show that, facing a capacity constraint, the supplier may preserve its monopoly power in equilibrium. This result stands in sharp contrast to the standard result that the supplier cannot preserve its monopoly power, which holds under the usual implicit assumption of an infinite production capacity.

1 Introduction

It is a well-established result in the literature on vertical contracting that when an upstream monopolist (the ‘supplier’) contracts bilaterally and privately with two (or more) competing downstream firms (the ‘retailers’), it is not able to preserve its market power and induce the monopoly outcome on the downstream market (Hart and Tirole (1990), McAfee and Schwartz (1994), Segal (1999), Rey and Tirole (2007)). This results from the monopolist’s inability to commit to a given set of contracts, which is a consequence of the assumption that offers are secret. With public offers, the monopolist is able to eliminate competition between retailers through manipulations of wholesale prices, typically raising these prices. However, the equilibrium set of public contracts is not an equilibrium of the game with secret contracts because the monopolist has an incentive to deviate and offer to one of the downstream firms a lower wholesale price. While the assumption of public contracts may make sense in some specific situations, for example because the law prohibits offering different contracts to different firms, it is in general difficult to justify. So, we are left with this uncomfortable

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result of a total inability for the monopolist to preserve its market power. This result is uncomfortable essentially because its robustness is questionable.

The first reason why the robustness of this result is questionable is that vertical contracting games typically have many equilibria. Due to the secrecy of offers, these are games of incomplete information and we thus have to consider (weak) perfect Bayesian equilibria (Mas-Colell, Whinston, Green (1995)). In a PBE, there is no constraint on out of equilibrium beliefs.¹ There are many possible out of equilibrium beliefs and, as a consequence, there are many PBE, even if not any system of beliefs can be part of an equilibrium. The equilibrium discussed above, in which the monopolist is unable to preserve its market power is in fact the PBE when retailers have passive beliefs.² Alternatively, one can assume that retailers have symmetry beliefs and then the monopolist is able to eliminate competition on the final market, just as in the game with public offers.

The choice of out of equilibrium beliefs is thus critical. It is also a difficult choice because we don’t know much about the way retailers think about the discussions between the supplier and their competitors.³ There is however a fruitful theoretical approach to this issue, first proposed by McAfee and Schwartz (1994), then reformulated by Rey and Vergé (2004). Rather than choosing beliefs, these authors propose a criterion that acceptable beliefs should satisfy and that allows them to select beliefs. They call these beliefs ‘wary beliefs’. If a system of beliefs can be shown to correspond to the wary beliefs in a given game, it is reasonable to assume that this system of beliefs holds in equilibrium.

Let us now consider the second reason why the robustness of this result is questionable. The result depends very much on some of the assumptions underlying the game. In particular, the monopolist is implicitly assumed not to face any capacity constraint. As noted by Rey and Tirole (2007), a tight capacity constraint may allow the monopolist to reduce the output on the final market and come closer to the monopoly outcome. However, neither Rey and Tirole (2007) nor any other contribution I am aware of work out the resolution of a vertical contracting game with a capacity constrained supplier.

In this article, I consider a vertical contracting game with a capacity constrained supplier. Introducing a capacity constraint in the model makes it necessary to fully reconsider the issue of out of equilibrium beliefs since passive beliefs cannot be assumed any more.⁴ I consider ‘full capacity beliefs’ that are consistent with an upstream capacity constraint. I demonstrate that these beliefs are wary beliefs when the capacity constraint is tight or production is costless. In the first case, the supplier’s ability to preserve its monopoly power in equilibrium depends on its production capacity. The preservation of monopoly power is achieved when the supplier’s production capacity is equal to the monopoly

¹They have to be consistent with strategy sets: players should not believe other players to make moves that they cannot make.
²As pointed at by McAfee and Schwartz (1995), this leads to the condition of pairwise-proofness imposed by Cremer and Riordan (1987) and O’Brien and Shaffer (1992).
³This is certainly a point on which experimental economics may be very helpful (Martin et al. (2001)).
⁴Symmetry beliefs cannot either be assumed when the supplier faces a capacity constraint.
output. In the second case, a sufficient condition for the supplier to preserve its monopoly power is that its production capacity is larger than or equal to the monopoly output.

The assumption of costless production may seem quite strong. However, many industries have recently been profoundly transformed by a process of dematerialization. The assumption is not so far from reality for at least some of these industries. For a firm selling music through the internet, the capacity constraint is a more important technical data than the marginal production cost, which is negligible.

In section 2, I present the game and define full capacity beliefs. In section 3, I present the PBE of the game when the supplier faces a tight capacity constraint and retailers have full capacity beliefs. I then show that these equilibria are wary beliefs equilibria of the game. In section 4, I consider the case of costless production and show that the PBE of the game with full capacity beliefs are wary beliefs equilibria. In both cases, I determine the conditions under which the supplier is able to preserve its monopoly power. Section 5 concludes.

2 The game

I consider an industry composed of a monopolistic supplier \((S)\) selling its product to final consumers through independent retailers. There are two retailers \((R_1\) and \(R_2)\) competing on the final market à la Cournot. The demand on the final market is assumed to be linear, with \(P(X) = 1 - X\).

\(S\) supports a constant marginal cost of production equal to \(c = 0\) for \(Q \leq \bar{Q}\) and \(c = +\infty\) for \(Q > \bar{Q}\). The supplier’s production capacity, \(\bar{Q}\), is finite, exogenous and it is public knowledge. Retailing costs are assumed to be null.

I denote the supplier’s profit by \(\Pi^S\) and industry profits, i.e. the joint-profits of \(S, R_1\) and \(R_2\), by \(\Pi\). For \(X \leq \bar{Q}\), \(\Pi(X) = (P(X) - c)X\).

Following Segal (1999), I consider a two stage game \(\Gamma(\bar{Q}, c)\). The timing of moves is:

Stage 1 (offers): \(S\) makes each retailer an offer that is observed only by this retailer. \(R_i\) is offered a pair \((q_i, t_i)\) where \(q_i\) is the quantity supplied by \(S\) and \(t_i\) is the transfer from \(R_i\) to \(S\).

Stage 2 (acceptances and competition): Each retailer decides whether to accept or reject the offer it received from the supplier. If \(R_i\) accepts the contract \((q_i, t_i)\), it pays \(t_i\), receives \(q_i\), and puts \(x_i \leq q_i\) on the market.\(^5\)

To maximize industry profits, the output on the final market should be equal to \(Q^*(c) = \frac{1}{2+c}\). We will refer to this output level as the ‘monopoly output’. It is indeed the output that a fully integrated monopolist would select. It is also the equilibrium of the public contracting game. \(S\) can for example offer \((\frac{Q^*(c)}{2}, \frac{\Pi^*(c)}{2})\), with \(\Pi^*(c) = \Pi(Q^*(c))\), to each retailer. Both accept and put

\(^5\)The retailing sector is a Cournot duopoly in which firms have a capacity constraint: they cannot sell more than the quantity they purchased from the monopolist. There is however the possibility that they don’t put on the market all the quantity purchased from the monopolist. This is why in general, \(x_i \leq q_i\).
\[\begin{align*}
x &= \frac{Q_i^c(c)}{2},
\end{align*}\] on the final market. Industry profits are maximized and captured by the supplier which in this case is able to preserve its monopoly power despite the fact that its product is retailed by a duopoly.

In the private contracting game with infinite production capacity and passive beliefs, offering \( \left( \frac{Q_i^c(c)}{2}, \frac{\Pi_i^c(c)}{2} \right) \) to each retailer is not an equilibrium. \( S \) can profitably deviate from this strategy by offering one of the retailers a slightly larger quantity for a slightly larger transfer. The retailer will accept the offer. The other retailer will still accept \( \left( \frac{Q_i^c(c)}{2}, \frac{\Pi_i^c(c)}{2} \right) \), which will lead to a loss for this retailer, and the monopolist will enjoy larger profits. In equilibrium, \( S \) offers each retailer \( \left( \frac{Q_i^c(c)}{2}, \frac{1-Q_i^c(c)}{2}Q_i^c(c) \right) \), where \( Q_i^c(c) = \frac{2}{3}(1-c) \) is the equilibrium aggregate output of a Cournot duopoly with marginal cost equal to \( c \). Both retailers accept the offer and put \( q_i^c = \frac{Q_i^c(c)}{2} \) on the market. Industry profits are reduced as compared to the previous case due to the supplier’s inability to preserve its monopoly power.

Let us now consider the private contracting game with a finite production capacity. \( \Gamma(Q, c) \) is an extensive form game of imperfect information. The resolution strategy is to look for the (weak) perfect Bayesian equilibria (PBE) of this game (Mas-Colell, Whinston, Green (1995)). A PBE is an assessment resolution strategy to look for the (weak) perfect Bayesian equilibria (PBE) \( \Gamma(Q, c) \).

I assume that \( S \) cannot offer more than \( Q \). The supplier’s strategy set is thus \( \mathcal{S} := \left\{ (q_1, t_1, q_2, t_2) \in (\mathbb{R}^+)^4 : q_1 + q_2 \leq Q \right\} \). A strategy for retailer \( R_i \) is a function \( x_i \) from \([0, Q]\) into \( \mathcal{Q} \cup \mathbb{R}^+ \), where \( x_i = \mathcal{Q} \) means that \( R_i \) rejects the offer and \( 0 \leq x_i \leq q_i \) means that it accepts the offer and puts the quantity \( x_i \) on the market. \( R_i \) holds beliefs \( \mu_i \). Given \( (q_i, t_i) \), \( \mu_i \) specifies a distribution of probability on the set of admissible values of \( (q_j, t_j) \). I restrict my attention to beliefs that put probability one on a particular contract, denoted by \( (\tilde{q}_i(q_i, t_i), \tilde{t}_i(q_i, t_i)) \). Beliefs must be compatible with the supplier’s strategy set, that is, \( (q_i, t_i, \tilde{q}_i(q_i, t_i), \tilde{t}_i(q_i, t_i)) \in \mathcal{S} \). This condition disqualifies passive beliefs as well as symmetry beliefs. Passive beliefs are in fact constant beliefs. Regardless of \( (q_i, t_i) \), \( R_i \) believes that, with probability one, \( R_j \) is offered the same contract \( (q_j^*, t_j^*) \). Now suppose that \( R_i \) is offered a contract specifying \( q_i = \overline{Q} \). \( R_i \) has no choice but to believe that \( R_j \) is offered \( q_j = 0 \). Passive beliefs are acceptable only if each retailer believes that the other retailer receives \( q = 0 \). This will certainly not be part of a PBE. Under symmetry beliefs, each retailer believes that the other retailer receives exactly the same offer as he does. The same reasoning leads to the disqualification of symmetry beliefs.

I propose to consider ‘full capacity beliefs’.

**Definition 1** Under full capacity beliefs, when \( R_i \) receives \( (q_i, t_i) \), it believes...
that $R_j$ is offered $\bar{q}_j = \bar{Q} - q_i$. As regards the transfer $t_j$, $R_i$ believes that $S$ sets $\hat{t}_j$ so as to leave no rent to $R_j$ given that $R_j$ has full capacity beliefs.

Obviously, full capacity beliefs are compatible with the supplier’s strategy set. The following sections characterize the PBE of $\Gamma(\bar{Q}, c)$ under full capacity beliefs and compare it with the PBE under wary beliefs in two particular cases.

3 Tight production capacity constraint

When the supplier’s capacity constraint is tight, we can expect that retailers don’t retain product in equilibrium, but rather put on the market all the product they receive from the supplier. This intuition is correct if we define a tight capacity constraint by the two following conditions. First, $\bar{Q} \leq 1 - c$. This ensures that industry profits are non-negative when $S$ supplies retailers with $\bar{Q}$ and retailers put $\bar{Q}$ on the market. Second, $\bar{Q} \leq \bar{Q}^*(0)$. If $R_i$ receives $\bar{Q} > \bar{Q}^*(0)$, it keeps $\bar{Q} - \bar{Q}^*(0)$ out of the market because $\bar{Q}^*(0)$ is the output of a monopolistic retailer.

In this section, I demonstrate that there exists perfect Bayesian equilibria with full capacity beliefs for the private contracting game $\Gamma(\bar{Q}, c)$ when the capacity constraint is tight. Furthermore, I characterize the set of perfect Bayesian equilibria with full capacity beliefs and show that full capacity beliefs are wary beliefs. I also show that, for $\bar{Q} = \bar{Q}^*(c)$, industry profits are maximized and entirely captured by the supplier. This means that private contracting is not in itself the origin of the loss of market power for a supplier that sells its product through independent retailers. The origin of this result is the combination of privacy, passive beliefs and infinite production capacity.\(^6\) The existence of a capacity constraint disqualifies passive beliefs and, under full capacity beliefs, which are also wary beliefs, the supplier may preserve its monopoly power. It will certainly do so if it is able to choose $\bar{Q}$.

I first define a family of assessments in which retailers hold full capacity beliefs.

**Definition 2** For any $\alpha \in [0, 1]$, $E_\alpha(\bar{Q}) = \{x_1, x_2, q_1, t_1, q_2, t_2, \bar{q}_1, \bar{t}_1, \bar{q}_2, \bar{t}_2 \}$, where:

1. $(q_1, t_1, q_2, t_2) = (\alpha \bar{Q}, \alpha(1 - \bar{Q})\bar{Q}, (1 - \alpha)\bar{Q}, (1 - \alpha)(1 - \bar{Q})\bar{Q})$

2. For $i \in \{1, 2\}$, $x_i = q_i$ for $t_i \leq (1 - \bar{Q})q_i$, $x_i = \emptyset$ otherwise and $((\bar{q}_i(q_i, t_i), \bar{t}_i(q_i, t_i)) = (\bar{Q} - q_i, (1 - \bar{Q}))(\bar{Q} - q_i))$.

In $E_\alpha(\bar{Q})$, retailers either reject the contract or put on the market all the product they receive from the supplier. The supplier produces at full capacity and shares the product between the retailers. Regardless of $\alpha$, the output on the

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\(^6\)Recall that with infinite production capacity and symmetry beliefs, the supplier is able to preserve its monopoly power. However, symmetry beliefs are impossible when the supplier’s production capacity is finite.
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Proposition 2 is in fact not the case. a family of equilibria, there could be other full capacity beliefs equilibria. This E

Proof. I prove the proposition by checking the sequential rationality of strategies and the consistency of beliefs.

Consistency of beliefs ((\tilde{q}_j(q_i, t_i), \tilde{t}_j(q_i, t_i))) is deduced from strategies through Bayes’ rule when (q_1, t_1, q_2, t_2) = (\alpha \bar{Q}, \alpha(1 - \bar{Q})\bar{Q}, (1 - \alpha)\bar{Q}, (1 - \alpha)(1 - \bar{Q})\bar{Q}).

Sequential rationality for R_i Strategy x_i should be optimal for R_i given other players’ strategies and the system of beliefs. Being offered (q_i, t_i), R_i believes that R_j is offered ((\tilde{q}_j(q_i, t_i), \tilde{t}_j(q_i, t_i)) = (\bar{Q} - q_i, (1 - \bar{Q})(\bar{Q} - q_i)). If R_j is indeed offered this contract, x_j = \bar{Q} - q_i because \tilde{t}_j \leq (1 - \bar{Q})\tilde{q}_j. Because of that, R_i’s profit will be (1 - \bar{Q} + q_i - x_i)x_i - t_i if it accepts the contract and puts x_i on the market. As a consequence, R_i puts q_i on the market if and only if q_i \leq ArgMax [(1 - \bar{Q} + q_i - x_i)x_i] \iff q_i \leq 1 - \bar{Q}, which holds for q_i \leq \bar{Q} \leq Q^*(0). If retailer R_i accepts the contract, it puts the quantity q_i on the market and makes a profit equal to (1 - \bar{Q})q_i - t_i. R_i accepts the contract if and only if t_i \leq (1 - \bar{Q})q_i. The strategy x_i is sequentially rational in E_\alpha(\bar{Q}).

Sequential rationality for S Assume that S deviates from (q_1, t_1, q_2, t_2) and offers (q'_1, t'_1, q'_2, t'_2) \in S. Setting transfers at the highest values that retailers can accept, S gets (1 - \bar{Q} - c) (q'_1 + q'_2). For \bar{Q} \leq 1 - c, profits are maximized over S for q'_1 + q'_2 = \bar{Q}, as in E_\alpha(\bar{Q}). There is no profitable deviation from E_\alpha(\bar{Q}) for the supplier.

Because the capacity constraint is tight, retailers are always capacity constrained in the sense that for any q_i, the best reply of R_i to \tilde{q}_j is larger than q_i. As a consequence, there is no product retention by retailers. As regards the supplier, changing the transfers without changing the quantities cannot be profitable because, in E_\alpha(\bar{Q}), no rent is left to retailers. Changing quantities and adapting transfers optimally to the new quantities leads to E_\alpha(\bar{Q}) for some \alpha’. This has no impact on profits.

While proposition 1 demonstrates the existence of equilibria and describes a family of equilibria, there could be other full capacity beliefs equilibria. This is in fact not the case.

Proposition 2 For any \bar{Q} \leq \min(Q^*(0), 1 - c) and any perfect Bayesian equilibrium with full capacity beliefs E of \Gamma(\bar{Q}, c), there exists \alpha \in [0, 1] such that E = E_\alpha(\bar{Q}).

Proof. In any full capacity beliefs equilibrium, the supplier produces at full capacity and shares its production between the retailers. As a consequence, (q_1, q_2) = (\alpha \bar{Q}, (1 - \alpha)\bar{Q}) for some \alpha \in [0, 1]. Sequential rationality constraints imply that E_\alpha is the unique full capacity beliefs equilibrium such that (q_1, q_2) = (\alpha \bar{Q}, (1 - \alpha)\bar{Q}). In particular, (t_1, t_2) \neq (\alpha(1 - \bar{Q})\bar{Q}, (1 - \alpha)(1 - \bar{Q})\bar{Q}) would
violate the supplier’s sequential rationality condition, either because it leads to rejection by one of the retailers or because it leaves a rent to one of the retailers.

Propositions 1 and 2 show that the set of perfect Bayesian equilibria with full capacity beliefs for \( \Gamma(\overline{Q}, c) \) is exactly the set of the assessments \( E_\alpha \) for \( \alpha \in [0, 1] \). This allows us to establish two results that are valid for any full capacity beliefs PBE of \( \Gamma(\overline{Q}, c) \).

**Corollary 1** For any \( \overline{Q} \leq \min(Q^*(0), 1 - c) \), the supplier’s profit is \( \Pi^S = (P(\overline{Q}) - c)\overline{Q} \) in any perfect Bayesian equilibrium with full capacity beliefs of \( \Gamma(\overline{Q}, c) \).

**Proof.** \( \forall \alpha \in [0, 1] \), in \( E_\alpha \), \( x_1 + x_2 = q_1 + q_2 = \overline{Q} \) and \( t_1 + t_2 = (1 - \overline{Q})\overline{Q} \). \( \Pi^S = t_1 + t_2 - c(q_1 + q_2) = (P(\overline{Q}) - c)\overline{Q} \). \( \blacksquare \)

**Corollary 2** When \( \overline{Q} = Q^*(c) \), \( \Pi^S = \Pi^*(c) \) in any full capacity beliefs perfect Bayesian equilibrium of \( \Gamma(\overline{Q}, c) \).

**Proof.** \( Q^*(c) \leq \min(Q^*(0), 1 - c) \) and \( \Pi^*(c) = (P(Q^*(c)) - c)Q^*(c) \). \( \blacksquare \)

In equilibrium, retailers put all the product they get from the supplier on the market. So, the output on the final market is \( X = \overline{Q} \). Through the transfers, the supplier captures downstream profits and \( \Pi^S = (P(\overline{Q}) - c)\overline{Q} \). \( \Pi^S \) is first increasing in \( \overline{Q} \), and then decreasing, with a maximum reached at \( \overline{Q} = Q^*(c) \). Eventually, \( \Pi^S \) may be lower than the profit the supplier would make in the absence of a capacity constraint under passive beliefs, that is, \( (P(Q^C(c)) - c)Q^C(c) \). This happens for \( Q^C(c) \leq \overline{Q} \leq \min(1 - c, Q^*(0)) \), which requires \( c \geq 1/4 \). For \( \overline{Q} = Q^*(c) \), \( \Pi^S \) is equal to \( \Pi^* \): The supplier preserves its monopoly power when its production capacity is equal to the monopoly output. This is a direct consequence of the existence of a full capacity beliefs PBE in which retailers put all the product on the market. This existence is proved by proposition 1. Proposition 2 further proves that there are no other full capacity beliefs PBE. However, there may be other PBE relying on other beliefs. This raises the question of whether the assumption of full capacity beliefs is convincing. I now show that full capacity beliefs are wary beliefs, as defined in Rey and Vergé (2004).

**Proposition 3** For any \( \overline{Q} \leq \min(Q^*(0), 1 - c) \), any full capacity beliefs perfect Bayesian equilibrium of \( \Gamma(\overline{Q}, c) \) is a wary beliefs perfect Bayesian equilibrium of \( \Gamma(\overline{Q}, c) \).

**Proof.** For \( E_\alpha \) to be a wary beliefs perfect Bayesian equilibrium of \( \Gamma(\overline{Q}, c) \), \( (\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (\overline{Q} - q_i, (1 - \overline{Q})(\overline{Q} - q_i)) \) must be profit maximizing for \( S \) for any admissible \( (q_i, t_i) \). By definition of full capacity beliefs, \( S \) leaves no rent to \( R_j \). If \( S \) is not maximizing profits, this can only be because the quantity offered is not optimal. Since \( S \) cannot offer more than \( \hat{q}_j = \overline{Q} - q_i \), the question is whether it may be profitable for \( S \) to offer a lower quantity to \( R_j \) and adjust the transfer to extract all the rent from \( R_j \). If \( S \) offers a
quantity \( q_j < Q - q_i \) to \( R_j \). \( R_j \) believes that \( R_i \) receives \( Q - q_j \). Given \( R_i \)'s strategy, it means that \( R_i \) will put on the market the quantity \( Q - q_j \). \( R_j \), if it accepts the contract, will choose \( x_j \) so as to maximize \((1 - Q + q_j - x_j)x_j\). As shown before, it is optimal for \( R_j \) to choose \( x_j = q_j \) for \( Q \leq Q^*(0) \). \( R_j \)'s gross profit is \((1 - \overline{Q})q_j\) and \( S \) extracts this profit by setting \( t_j = (1 - \overline{Q})q_j \). Thus, \( \Pi^S = (1 - \overline{Q} - c)(q_j + q_j) \). Given \( q_i \), the supplier’s profit maximizing offer to \( R_j \) is clearly \((q_j, t_j) = (Q - q_i, (1 - Q)(\overline{Q} - q_i)) = ((q_j(q_i, t_i), t_j(q_i, t_i)) \) for \( c \leq 1 - \overline{Q} \).

When the supplier offers \( q_i \) to \( R_i \), it has no incentive to offer \( q_j < \overline{Q} - q_i \) to \( R_j \). Indeed, this would induce \( R_j \) to believe that \( R_i \) was offered more than \( q_i \). \( R_j \) accepts to pay a larger transfer when it receives \( \overline{Q} - q_i \) and believes that \( R_i \) receives \( q_i \) than when it receives \( q_j < \overline{Q} - q_i \) and believes \( R_i \) receives more than \( q_i \). This is why full capacity beliefs are wary beliefs when the capacity constraint is tight. This result provides a strong support to the assumption that retailers hold full capacity beliefs.

While I don’t solve the model for a loose capacity constraint and a strictly positive upstream marginal cost, it is possible to give some insights on what happens in this case. To clarify, let us assume that \( \overline{Q} \) is large and \( c > 0 \). Then, if the supplier produces at full capacity and sells all of its output to retailers, at least one of the retailers will find it optimal to put on the market only a fraction of the quantity received from the supplier. Part of the supplier’s output stays out of the market and thus generates no revenue, while producing it is still costly for the supplier. So, when \( \overline{Q} \) is above a certain threshold, an increase in \( \overline{Q} \) will reduce the supplier’s profit. At the extreme, the supplier would make a loss at a PBE with full capacity beliefs, which means that there is no such equilibrium. An equilibrium with full capacity beliefs may not exist when the capacity constraint is loose. Furthermore, full capacity beliefs may not coincide with wary beliefs. I leave for future work the question of the exact range of values of \( \overline{Q} \) over which our propositions in the general case are valid, but in the next section, I deal with this issue in the special case in which production is costless \((c = 0)\).

4 Costless production

The discussion above indicates that the non-existence of a PBE with full capacity beliefs when the capacity constraint is loose is related to upstream production costs. This suggests that the range of value of \( \overline{Q} \) for which such an equilibrium exists may increase when upstream marginal costs decrease. In this section, I make the extreme assumption that production is costless. Assuming \( \overline{Q} > Q^*(0) \), since the results from section 3 apply to \( \overline{Q} \leq Q^*(0) \), I show that a PBE with full capacity beliefs exists. I characterize the set of perfect Bayesian equilibria with full capacity beliefs and show that, in any of these PBE, the supplier maximizes and captures industry profits. I also show that full capacity beliefs are wary beliefs. In this section, I drop the argument of functions when it is equal to
zero. For example, $Q^*$ denotes $Q^*(0).$

The supplier is able to produce more than the output of a monopolistic retailer. This implies that there is some production retention by retailers. We will consider two ranges of values of $Q$, depending on whether $Q$ is smaller or larger than the output of a Cournot duopoly, $Q^C$. For $Q \in (Q^*, Q^C)$, it is useful to introduce two particular values of $q$. When $R_i$ is offered $(q_i, t_i)$, it believes that $R_j$ receives $Q - q_i$. As a consequence, $R_i$ would like to put on the market a quantity corresponding to its best reply to $Q - q_i$ in the Cournot game. Given that $R_i$’s best reply function is $BR(q_i) = \frac{1}{2}(1 - q_i)$, $R_i$ would like to play $\frac{1}{2}(1 - Q + q_i)$, which he can do only if $\frac{1}{2}(1 - Q + q_i) \leq q_i \iff q_i \geq 1 - Q$. I define $q’$ by $q’ = 1 - Q$. When a retailer receives a quantity above $q’$, it is able to play its best reply to what it believes its competitor receives. Part of the product received from the supplier stays out of the market. When a retailer receives a quantity below $q’$, it is constrained and puts on the market all the product it receives from the supplier. Consider the situation from the point of view of $R_j$. $R_j$ receives $Q - q_i$. $R_j$ believes that $R_i$ receives $q_i$ and that it is thus optimal to play $BR(q_i) = \frac{1}{2}(1 - q_i)$. $R_j$ will indeed do so if $\frac{1}{2}(1 - q_i) \leq Q - q_i \iff q_i \leq q''$, where $q'' = 2Q - 1$. Otherwise, $R_j$ will be constrained and put $Q - q_i$ on the market. Note that for $Q \in (Q^*, Q^C)$, $0 \leq q'' < q’$. These values appear in the following definition.

**Definition 3** For any $\alpha \in \{0, 1\}$, $E'_\alpha(Q) = \{ x_1, x_2, q_1, q_2, t_1, t_2, \ldots, \hat{q}_1, \hat{t}_1, \hat{q}_2, \hat{t}_2 \}$, where:

1. $(q_1, t_1, q_2, t_2) = (\alpha Q, \alpha \Pi^*, (1 - \alpha) Q, (1 - \alpha) \Pi^*)$

2. For $i \in \{1, 2\}$,

(a) $Q \in (Q^*, Q^C)$: For $0 \leq q_i \leq q’$, $x_i = q_i$ for $t_i \leq (1 - q_i - BR(q_i))q_i$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - q_i - BR(q_i))BR(q_i))$.

For $q’ \leq q_i \leq q’’$, $x_i = q_i$ for $t_i \leq (1 - Q)q_i$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - Q)(Q - q_i))$.

For $q’’ \leq q_i \leq Q$, $x_i = BR(Q - q_i) < q_i$ for $t_i \leq (1 - BR(Q - q_i) - Q + q_i)BR(Q - q_i)$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - Q + q_i - BR(Q - q_i))(Q - q_i))$.

(b) $Q > Q^C$: For $0 \leq q_i \leq q’$, $x_i = q_i$ for $t_i \leq (1 - q_i - BR(q_i))q_i$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - q_i - BR(q_i))BR(q_i))$.

For $q’ \leq q_i \leq Q - q^C$, $x_i = q^C$ for $t_i \leq (1 - 2q^C)q^C$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - 2q^C)q^C)$.

For $Q - q^C \leq q_i \leq Q$, $x_i = BR(Q - q_i) < q_i$ for $t_i \leq (1 - BR(Q - q_i) - Q + q_i)BR(Q - q_i)$, $x_i = \emptyset$ otherwise and

$((\hat{q}_j(q_i, t_i), \hat{t}_j(q_i, t_i)) = (Q - q_i, (1 - Q + q_i - BR(Q - q_i))(Q - q_i))$.

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$Q^* = \frac{1}{2}, Q^C = \frac{2}{3}, \Pi^* = \frac{1}{4}$ and $\Pi^C = \frac{1}{5}$.
Contrary to what happens in \(E_\alpha\), in \(E_0'\) and \(E_1'\), retailers may keep part of the product they receive from the supplier out of the market. For example, for \(Q > Q^C\), when \(R_1\) receives \(q^C\) and \(R_2\) receives \(q^C + \varepsilon < Q - q^C\), both retailers put \(q^C\) on the market and \(R_2\) keeps \(\varepsilon\) out of the market. When \(R_1\) receives \(Q\), it believes that \(R_2\) receives zero and, thus, plays its best reply to zero, which is just the monopoly output, \(Q^*\). Again, this is less than \(Q\). The proposition establishes that this last situation is precisely what happens in equilibrium.

**Proposition 4** For \(Q > Q^*\), \(\Gamma(Q, 0)\) has two perfect Bayesian equilibria with full capacity beliefs, \(E_0'\) and \(E_1'\). The output on the final market is \(Q^*\).

**Proof.**

1) \(Q \in (Q^*, Q^C]\)

**Sequential rationality of \(R_i\)** For \(0 \leq q_i \leq q''\), \(\bar{q}_j \geq q'\): Given \(R_j\)'s strategy \(\bar{t}_j(q_i, t_1)\), \(R_j\) accepts the contract and plays \(x_j = BR(R(Q - \bar{q}_j)) = BR(q_i) = \frac{1}{2}(1 - q_i)\). \(x_i = \min(BR(BR(q_i)), q_i) = q_i \iff t_i \leq (1 - q_i - BR(q_i))q_i\). For \(q'' \leq q_i \leq q'\), \(\bar{q}_j \in [q''', q']\), \(x_j = \bar{q}_j\) and \(x_i = \min(BR(Q - q_i), q_i) = q_i \iff t_i \leq (1 - q_i - BR(q_i))q_i\). For \(q \leq q_i \leq Q\), \(0 \leq \bar{q}_j \leq q''\). \(x_i = \bar{q}_j\) and \(x_i = \min(BR(Q - q_i), q_i) = BR(Q - q_i) \iff t_i \leq (1 - BR(Q - q_i) - Q + q_i)BR(Q - q_i)\).

**Consistency of \(R_i\)'s beliefs** Full capacity beliefs are consistent with the supplier’s equilibrium strategy.

**Sequential rationality for \(S\)** Without loss of generality, I establish the proof for \(E_1'\) under the assumption that \(S\) makes acceptable offers to retailers.

In \(E_1'\), the supplier offers \(Q\) to \(R_1\) and \(\bar{q}_2 = 0\). Consequently, \(x_1 = BR(0) = Q^*\). \(\Pi^S = t_1 + t_2 = \Pi^S\). Let us now consider all the possible deviations and show that they cannot be profitable. If \(S\) offers \(q_1 \in [0, q''']\), it can offer any \(q_2 \in [0, Q - q_1]\), \((Q - q_1) > q'\). For \(q_2 \in [0, q''']\), \(S\) can charge \(R_1\) a transfer equal to \((1 - q_1 - BR(q_1))q_1 = \frac{1}{2}(1 - q_1)q_1\). \(S\) maximizes profits by offering \(q_1 = q_2 = q'''\) and gets \((1 - q'''')q'' < \Pi^*\). For \(q_2 \in [q''', q']\), \(S\) charges \(t_1 = \frac{3}{2}(1 - q_1)q_1\) and \(t_2 = (1 - Q)q_2\). Maximizing \(t_1 + t_2\), \(S\) gets \(\frac{3}{2}(1 - q''')q''' + (1 - Q)q' = (1 - Q)(2Q - 1) + (1 - Q)^2) = (1 - 2Q) < \Pi^*\). For \(q_2 \in [Q, Q - q_1]\), \(S\) gets \(t_2 = (1 - BR_2(Q - q_2) - Q + q_2)BR_2(Q - q_2) = \frac{1}{2}(1 - \overline{Q} + q_2)^2\). This is maximal for \(q_2 = \overline{Q} - q_1\). So, \(S\) chooses \(q_1\) in order to maximize \(\frac{3}{4}(1 - q_1)q_1 + \frac{3}{4}(1 - q_1)^2 = \frac{3}{4}(1 - q_1)^2\).

\(S\) offers \(q_1, q_2) = (0, \overline{Q}) \iff \Pi^S = \Pi^*\). If \(S\) offers any \(q_2 \in [0, \overline{Q} - q_1]\), \(q''' \leq \overline{Q} - q_1 \leq q'\). There is no profitable deviation with \(q_2 \in [0, q''']\) (see above by symmetry). For \(q_2 \in [q''', \overline{Q} - q_1]\), \(S\) charges \(t_1 = (1 - Q)q_1\) and \(t_2 = (1 - Q)q_2\) and gets \((1 - Q)(q_1 + q_2) < \Pi^*\). If \(S\) offers \(q_1 \in [q'', \overline{Q} - q_1]\), it can offer any \(q_2 \in [0, \overline{Q} - q_1]\), where \(\overline{Q} - q_1 \leq q''\). \(S\) maximizes \(t_1 + t_2 = \frac{1}{4}(1 - \overline{Q} + q_1)^2 + \frac{1}{2}(1 - q_2)q_2\). This requires that \(q_2 = \overline{Q} - q_1\), leading to \(t_1 + t_2 = \frac{1}{4}(1 - \overline{Q} + q_1)^2 + \frac{1}{2}(1 - \overline{Q} + q_1)(\overline{Q} - q_1) = \frac{1}{4}(1 - (\overline{Q} - q_1)^2\). This is maximal for \(q_1 = \overline{Q}\), as in \(E_1'\).

2) \(Q > Q^C\)

**Sequential rationality of \(R_i\)** For \(0 \leq q_i \leq q^C\), \(\bar{q}_j \geq Q - q^C\). \(x_j = BR(q_i)\) and \(x_i = \min(BR(BR(q_i)), q_i) = q_i \iff t_i \leq (1 - q_i - BR(q_i))q_i\). For \(q^C \leq
any deviation for the supplier and if marginal costs are low enough, it may be an equilibrium. With zero marginal costs, it is indeed
an equilibrium. For the supplier to be able to preserve its market power, it is thus not necessary that its production capacity is equal to the monopoly output when production is costless. The necessary condition is rather that the supplier is able to produce at least the monopoly output.

I now show that full capacity beliefs are vary.

**Proposition 5** For $\overline{Q} > Q^*$, any perfect Bayesian equilibrium with full capacity beliefs of $\Gamma(\overline{Q}, 0)$ is a perfect Bayesian equilibrium with vary beliefs of $\Gamma(\overline{Q}, 0)$.

**Proof.** This proof requires to show that, for any admissible $(q_i, t_i)$, $R_i$ maximizes profits when offering to $R_j$ the contract $((\tilde{q}_j, q_i), \tilde{t}_j(q_i, t_i))$.

1) $\overline{Q} \in (Q^*, Q_C]$ 
   For $q_i \in [0, \overline{Q} - q']$, $q_j \in [0, \overline{Q} - q_i]$, with $\overline{Q} - q_i \geq q'$. For $q_j \in [0, q'']$, $t_j = \frac{1}{2} (1 - q_j) q_j$. $S$ chooses $q_j = q''$ and gets $t_j = \frac{1}{2} (1 - q'' q'')$. For $q_j \in [q'', q']$, $t_j = (1 - \overline{Q}) q_j$. $S$ chooses $q_j = q'$ and gets $t_j = (1 - \overline{Q}) q' = (1 - \overline{Q} q')$. 
   For $q_j \in [q', \overline{Q} - q_i]$, $t_j = \frac{1}{4} (1 - \overline{Q} q_j)^2$. $S$ chooses $q_j = \overline{Q} - q_i$ and gets $t_j = \frac{1}{4} (1 - q_i q_j)^2$. Since $\frac{1}{2} (1 - q'' q'') \leq (1 - \overline{Q})^2 \leq \frac{1}{2} (1 - q_i)^2$, $S$ maximizes profits by offering $q_j = \overline{Q} - q_i$ and charging the highest acceptable transfer $t_j(q_i, t_i)$. For $q_j \in [\overline{Q} - q', \overline{Q} - q'']$, $q_j \in [0, \overline{Q} - q_i]$, with $q'' \leq \overline{Q} - q_i \leq q'$. For $q_j \in [0, q'']$, $t_j = \frac{1}{2} (1 - q_j) q_j$. $S$ chooses $q_j = q''$ and gets $t_j = \frac{1}{2} (1 - q'' q'')$. 
   For $q_j \in [q'', \overline{Q} - q_i]$, $t_j = (1 - \overline{Q}) q_j$. $S$ chooses $q_j = \overline{Q} - q_i$ and gets $t_j = (1 - \overline{Q}) (\overline{Q} - q_i)$. Since $\frac{1}{2} (1 - q'' q'') \leq (1 - \overline{Q}) (\overline{Q} - q_i)$, $S$ maximizes profits by offering $q_j = \overline{Q} - q_i$ and charging the highest acceptable transfer $t_j(q_i, t_i)$. 
   For $q_i \in [\overline{Q} - q', \overline{Q} - q'']$, $q_j \in [0, \overline{Q} - q_2] \subset [0, q'']$. $t_j = \frac{1}{2} (1 - q_j) q_j$, which is maximal for $q_j = \overline{Q} - q_i$.

2) $\overline{Q} > Q^*$ 
   For $q_i \in [0, q^C]$, $q_j \in [0, \overline{Q} - q_i]$, with $\overline{Q} - q^C \leq \overline{Q} - q_i \leq \overline{Q}$. For $q_j \in [0, q^C]$, $t_j = \frac{1}{2} (1 - q_j) q_j$. $S$ chooses $q_j = q^C$ and gets $t_j = \frac{1}{2} (1 - q^C q^C)$. For $q_j \in [q^C, Q - q^C]$, $t_j = (1 - 2 q^C q^C)$. For $q_j \in [\overline{Q} - q^C, \overline{Q} - q_i]$, $t_j = \frac{1}{4} (1 - \overline{Q} + q_i)^2$. $S$ chooses $q_j = \overline{Q} - q_i$ and gets $t_j = \frac{1}{4} (1 - q_i)^2$. Since $\frac{1}{4} (1 - q^C q^C) = (1 - 2 q^C q^C) \leq (1 - q_i)^2$, $q_j = \overline{Q} - q_i$.
   For $q_j \in [q^C, \overline{Q} - q^C]$, $q_j \in [0, \overline{Q} - q_i]$, with $q^C \leq \overline{Q} - q_i \leq \overline{Q} - q^C$. For $q_j \in [0, q^C]$, $t_j = \frac{1}{2} (1 - q_j) q_j$. $S$ chooses $q_j = q^C$ and gets $t_j = \frac{1}{2} (1 - q^C q^C)$. For $q_j \in [q^C, \overline{Q} - q_i]$, $t_j = (1 - 2 q^C q^C)$. Since $\frac{1}{2} (1 - q^C q^C) = (1 - 2 q^C q^C)$, $S$ is indifferent as regards the value of $q_j$. In particular, $q_j = \overline{Q} - q_i$ maximizes $S$’s profit. For $q_i \in [\overline{Q} - q'', \overline{Q}]$, $q_j \in [0, \overline{Q} - q_i] \subset [0, q'']$ and $t_j = \frac{1}{2} (1 - q_j) q_j$, which is maximal for $q_j = \overline{Q} - q_i$.

The intuition for this result is the same as for a tight capacity constraint. The only difference is that, because the supplier’s production capacity is large, the supplier may be indifferent between offering $R_j$ a quantity equal to the production capacity available after its offer to $R_i$ and producing below capacity. Indeed, as long as both retailers are offered more than the Cournot duopoly output, they are indifferent regarding a variation in the quantity offered to them or to their competitor.
5 Conclusion

This article contributes to the literature in two ways. It exposes the source of a standard result, namely that a monopolist supplying retailers through secret contracts under passive beliefs as well as under wary beliefs is unable to preserve its monopoly power. This result is a consequence of the implicit assumption that the monopolist enjoys an infinite capacity of production. The article develops a theoretical framework to analyze vertical contracting in the presence of a finite production capacity at the upstream level. While the article doesn’t present a resolution of the game for every value of production capacity and upstream marginal cost, it presents a full resolution in two significant cases. In both cases, I characterize the (non-empty) set of perfect Bayesian equilibria of the game in which retailers hold full capacity beliefs, a type of beliefs that is discussed here for the first time. I further show that these full capacity beliefs correspond, in these cases, to the wary beliefs introduced by McAfee and Schwartz and reformulated by Rey and Vergé.

With costless production, as soon as the monopolist’s production capacity is sufficiently large, but finite, the monopolist can extend its monopoly power into the final market, maximize industry profits and entirely capture these profits.

With costly production, the supplier maximizes and captures industry profits when the upstream capacity is equal to the monopoly output. A direct consequence of this result is that if the monopolist is able to choose its production capacity, it will be able to monopolize the final market.

I leave for future work the characterization of wary beliefs and full capacity beliefs perfect Bayesian equilibria when upstream marginal costs are strictly positive and the capacity constraint is loose. This characterization is not necessary to establish, as I do in this article, that the standard model of private vertical contracting includes an implicit assumption (infinite upstream production capacity) that is the source of a supplier’s inability to preserve its market power when selling its product through competing retailers. Incorporating a capacity constraint into the model leads to very different results and certainly improves the predictive power of the model for many industries.

6 References


McAfee, R.P. and M. Schwartz, 1994, Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity, American Econ-

