A GENERALISED INDEX OF COMPETITIVE BALANCE IN PROFESSIONAL SPORTS LEAGUES

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1. Introduction

The question of Competitive Balance (CB henceforth) in Professional Sports leagues is still one of the most essential in the sport economics literature. In the case of European football, the majors events of the twenty last years (the emergence of the champions league, the Bosman decision, the burst of the broadcasting rights,. . .) all potentially distort it and many authors have analysed their consequences on the league revenues, the fan attendances and even the intrinsic beauty of the game (Vrooman (2007)). It is traditionally considered that, in a perfectly balanced league, each team has the same probability of winning so that there is a large uncertainty on what will be the result of the championship[1]. Conversely, an unbalanced competition is supposed to depress fan interest, demand and confidence in the legitimacy of the competition (Neale (1964), Daly (1992), Zimbalist (2003) and Késenne (2006)). Szymanski (2001, 2007) qualifies this idea since he observed that, in some cases, the majority of the fans seems to prefer that a dominant team blows out all the opponents (like Manchester United in English Premier League (EPL) during the 90's) and that despite its growing imbalance since 1995, the attractiveness of the EPL has not failed. Nevertheless, it seems that in the long run, a persistently unbalanced contest would experience a decline of attendance. In the North American closed leagues, a bundle of mechanisms (revenue sharing agreements, salary caps, luxury taxes,. . .) is mobilised to ensure a sufficient degree of CB (Quirk and Fort (1999)). In the case of European leagues, the assumed win maximising behaviour of the clubs (Sloane (1977)) can damage CB since all surplus income is supposed to be reinvested in talents; on the other side, the promotion/relegation systems is supposed to mechanically increase CB by annual replacement of weaker teams[2]. But once more, as remarked by Vrooman (2007) or Szymanski (2007), the system of promotion and relegation in Europe seems to fail to produce CB in domestic leagues because it mainly reveals the different classes of teams ranked by their revenue-generating potential. Moreover, the events of the two past decades have favoured imbalance: if it is not clear that the Bosman decision has increased imbalance within the domestic leagues, Késenne (2007) establishes that it favoured it between them. As concern imbalance within the leagues, either the actual format of the Champions league (Késenne (2007), Vrooman (2007)) or the

[1] We focus here on "between seasons Competitive Balance" in the sense of Szymanski (2003)

[2] A study of the factors that increase the expected degree of competitive balance can be found in Noll (2002).

unequal sharing of television revenues (Noll (2007)) clearly intensify it. As a consequence, Késenne (2007), Szymanski (2007) and Vrooman (2007) propose the creation of a European Super League build on the solidarity model of the NFL. Vrooman (2007) argues for a 30 teams leagues, whereas Hoehn and Szymanski (1999) originally militate in favour of a 60 teams one.

The aim of this paper is not to analyse the optimal size of a future European Super League, but to propose a satisfying tool to measure CB within any league, and especially a tool capable of accounting for changes in its size. We show that the commonly used measures are not suitable and propose a Generalized Ratio of Competitive Balance. The most commonly used measure in the literature is probably the dispersion (standard deviation) of winning percentage[3] within the league (Scully (1989), Quirk and Fort (1999)). Dealing with some European leagues, especially football, the winning percentage of teams is not a relevant statistic because an important part of game's results are draws[4]. Naturally, the authors then rather focus on the percentage of points obtained by teams in the league. Consequently, the measure of CB is either established on the standard deviation of the percentage of points, or on a Hirshman Herfindahl Index (HHI), which is a concentration index calculated as the sum of squared shares of points won by each team in the league. Another important consequence of this feature of European leagues, is that the point award system may not only impact the behaviour of teams in each game but also the relevant way to measure competitive balance. This question is neither neutral on the question of the suitable measure of CB which account for a change in the size of leagues. Modifications in the size of leagues is not a secondary question in European football. Actually, in all of the 5 major European football leagues (England, France, Germany, Italy, Spain)[5], the size of the leagues has changed many times since 1960 (expansion or contraction): there were 6 changes in the size of the French league, 4 changes in the size of the Spanish league, 3 changes in the size of the English, German or Italian leagues; for instance, consequently to those changes, the Spanish league knew 4 different sizes (a league made of 16, 18, 20 or 22 teams)[6]. To account for changes in sizes, following Depken (1999), a Hirshman Herfindahl index of Competitive Balance has been developed, either by subtracting the minimal value of HHI (dHHI) or by constructing a ratio, placing the minimal value at the denominator (HICB) (Lenten (2009), Pawlowski et al. (2010)). According to Hall and Tideman (1967), concentration measures must satisfy a set of desirable axioms and in particular (axiom vi) a concentration index should be between zero and one. Unfortunately, neither the dHHI index proposed by Depken (1999), nor the HICB indexes (Lenten (2009), Pawlowski et al. (2010)) satisfy the upper bound to one. In this paper we propose a measure of CB which satisfies this desirable axiom. Moreover, a concentration index must also fulfil another property: the principle of transfers by which concentration should decrease if the share of any firm is decreased at the benefit of a smaller firm (see Hannah and Kay (1977)). In sport economics this property means that a reallocation of points at the expense of better ranked team implies a increase in CB. When the point system is such as the total number of points distributed in a league is not a constant, the

[3]There also exists papers where Competitive Balance is not measured on the actual results of the games but on the distribution of talents among clubs. See for instance Feess and Stähler (2009).

[4]More than 25 % of the matches' results are draws in European football. The historical focus of sport economics on Baseball, where no draw can arise, probably explains the very large preference for the winning percentage rather than the percentage of points obtained as the relevant variable.

[5]The case of the Scottish Premier League is much more complicated because it has not only operated an expansion of the number of teams but also created an unbalanced schedule. This has been examined by Lenten (2009).

[6]The rythm of change is even higher in the Major League Soccer in North America: between 1996 and 2010, the size of the league has changed seven times!

principle of transfers does not hold.

The paper is organized as follows: we start by applying the desirable properties of a concentration index to the measure of CB and shed the light on the importance of the design of the point award system. We then identify the hypothetical configuration that maximizes the value of any of such an index. This allows us to propose a family of measures of CB defined on the zero-one interval, called Generalized Ratio of Competitive Balance. Remarkably, the measures based on the Hirshman Herfindhal Index or on the standard deviation are perfectly identical. In the last section, we try to give a more robust foundation to the concept of CB with help of a probabilistic approach: in a perfectly balanced league, each team has the same probability of winning or loosing and this common probability is the same for any club. In a sport where draws can happen, it is relevant to focus on the probability of draw and study its distribution function. We can then build a criterion to systematically answer the question: was this league balanced or not? We apply this test on the 5 majors European football leagues between 1960 and 2010, and question the possible decline of CB since 1995, resulting to the Bosman case.

2. Competitive balance indices

In this section we begin by formalizing the distribution of points in a league and introduce various Competitive balance indices. Let n be the number of teams in a league. If team i wins against team j then team i obtains $\alpha \in \mathbb{N}$ points and team j 0 point. If the game ends up in a draw, each team is rewarded by $\beta \in \mathbb{N}$ points. Historically, the point award system in the European football has been unstable[7]. If one is willing to measure the impact of a change in the point award system on the Competitive Balance, the index must be defined on a unique basis. In the sequel we shall assume that the point award system is time invariant.

Proposition 1. *Given n, the total number of points distributed in a season is constant if and only if the time invariant point award system is such that $\alpha/\beta = 2$. The total number of points is then $\mathcal{T}_\beta(n) = 2\beta n(n-1)$.*

This proposition states that the total number of points in a league season is independent of the match results if and only if a victory is rewarded two times more than a draw. This is not surprising because under this condition the number of points distributed in each match is invariant with respect to the results. In the sequel, we will mainly focus on this case and assume, without loss of generality, that $\beta = 1$ and $\alpha = 2$. This restriction may be seen as arbitrary and without strong empirical support. It will become clear later that this restriction is required to build an index of competitive balance with desirable properties. Note that the 3-1-0 point award system, which is the actual system in european football, would not allow to build a CB index with desirable properties.

The competitive balance indices are based on the relative number of points accumulated by each team in a season[8]. Before turning to the definition of a CB index, we describe the relevant properties of the distribution of points.

Property 1. *Let $\mathbf{p} \equiv (p_1, p_2, \ldots, p_n)$ be a vector gathering the total number of points won by teams $i = 1, \ldots, n$ in a season. This vector $\mathbf{p}$ satisfies the following properties:*

[7]In France as in Germany, the 3-1-0 was definitively adopted in 1995, while the same system was already adopted since 1982 in England.

[8]In the standard schedule, where each team plays against any other twice (once home – once away).

(a) $p_i \geq 0 \ \forall i \in \{1, \ldots, n\}$,
(b) $\sum_{i=1}^{n} p_i = 2n(n-1)$,
(c) $\max_i p_i \leq 4(n-1)$,
(d) *If* $\exists\, i$ *such that* $p_i = 4(n-1)$ *then* $\max_{j \neq i} p_j \leq 4(n-2)$,
(e) *For all* $m \in \{1, \ldots, n\}$, *any m-uplet* $\left(p_{i_1}, p_{i_2}, \ldots, p_{i_m}\right)$ *is such that:*

$$\sum_{j=i_1}^{i_m} p_j \leq 2m(2n-m-1)$$

We denote $\mathscr{P}_n$ *the set of vectors in* $\mathbb{N}^n$ *satisfying (a)–(e).*

Property 1(a) is obvious and property 1(c) is a direct consequence of proposition 1. The following property, 1(b), defines the maximum number of points that a team can obtain in a season. This upper bound is strictly inferior to the total number of points distributed in a season (provided $n > 2$), because this particular team will not participate to all the games played in the season. 1(c) is established by counting the number of points that a team winning all the matches would obtain. Property 1(d) states that no more than one team can attain the upper bound identified in 1(c). Two teams A and B cannot win all their matches, because A meets B twice and the results in these two matches cannot be victory for both teams. If team A attains the upper bound defined in 1(c) at the end of the season, then team B can at most expect a total number of points equal to $4(n-1) - 4 = 4(n-2)$, because B "looses" 4 point against A. Finally property 1(e) generalises properties 1(c) and 1(d) by providing an upper bound for the points obtained by $m < n$ teams. One can easily show that the set $\mathscr{P}_n$ is not empty, for instance by establishing that the vector $\mathbf{p}$ with $p_i = 2(n-1)$ for all $i \in \{1, \ldots, n\}$ belongs to $\mathscr{P}_n$. This vector is associated to a season where all the games end up in draws.

For any economist, there is an obvious analogy between the market share of firms in an industry and the share of points won by each team in a league during a season. This analogy is not perfect since no monopoly can arise except in the degenerated case of a two teams league. However, the Hirschman Herfindahl index of concentration has interesting characteristics and is relevant to measure the degree of Competitive Balance in a league.

We identify the output of one team by the number of points obtained in a tournament. The market share of team i ($s_i(\mathbf{p})$), in a tournament composed of n rival teams, is defined as

$$s_i(\mathbf{p}) = \frac{p_i}{\sum_{i=1}^{n} p_i}$$

where p_i refers to the number of points obtained by a team ($0 \leq s_i(\mathbf{p}) < 1$). The Herfindahl-Hirschman index is defined as:

$$\mathrm{HHI}(\mathbf{p}) = \sum_{i=1}^{n} s_i(\mathbf{p})^2$$

In the case where all the teams of a league obtain the same number of points ($\sum_{i=1}^{n} p_i = np_j$ for all $j = 1, \ldots, n$), a configuration that we call Perfect Competitive Balance (PCB herafter), HHI is equal to $1/n$. Obviously, the variability of the lower bound has to be considered when different sizes of leagues are at stake.

To account for changes in the size of the leagues, Depken (1999) proposes to refer to an index constructed as the difference between the observed Hirschman Herfindahl index and its minimal value, which, as stated previously, is not constant when measuring the "concentration" of teams in a sport league. Depken (1999) suggests to use $\text{dHHI}(\mathbf{p}) = \text{HHI}(\mathbf{p}) - \min_{\mathbf{q}\in\mathscr{P}_n} \text{HHI}(\mathbf{q}) = \text{HHI}(\mathbf{p}) - \frac{1}{n}$. When a Hirshman Herfindahl index is measured in an industry, one can also refer to the "equivalent number" of firms n^e, that is to say, the number of firms with strictly identical market shares that would lead to the actual HHI. When transposed to the question of Competitive Balance, n^e is the hypothetical numbers of teams playing in a perfectly balanced league that would duplicate the observed value of HHI, *ie* $\text{HHI} = 1/n^e$. We have $n^e \leq n$ and equality in the case of a perfect competitive balance.

As an alternative, Lenten (2009) and Pawlowski et al. (2010) proposed the HICB, defined as a ratio instead of a difference:

$$\text{HICB}(\mathbf{p}) = \frac{\text{HHI}(\mathbf{p})}{\min_{\mathbf{q}\in\mathscr{P}_n} \text{HHI}(\mathbf{q})} = \frac{\text{HHI}(\mathbf{p})}{\frac{1}{n}} = n\sum_{i=1}^{n} s_i(\mathbf{p})^2$$

Unfortunately, both the dHHI and HICB indices are still sensitive to the size of the league because they do not account for the induced variations of the upper bound of the HHI index. Actually, this issue is specific to sport economics because, as mentionned earlier, even the leader cannot hold 100% of the market shares (provided $n > 2$). In standard IO application, if a monopole holds all the market shares the value the HHI index will be equal to one so that the maximum value of HHI index is one. In our context the maximum value of HHI is less than one and depends on the size of the league[9].

In this paper we consider an index such that the (effective) lower and upper bounds are 0 (Perfect Competitive Balance) and 1 (Perfect Hierachy or Perfect Competitive Imbalance). Henceforth, these bounds are not affected by the number of teams in a league.

Definition 1. *The HRCB index (Herfindahl Ratio of Competitive Balance) is defined as:*

$$\text{HRCB}(\mathbf{p}) = \frac{\text{HHI}(\mathbf{p}) - \min_{\mathbf{q}} \text{HHI}(\mathbf{q})}{\max_{\mathbf{q}} \text{HHI}(\mathbf{q}) - \min_{\mathbf{q}} \text{HHI}(\mathbf{q})}$$

By construction, this index belongs to $[0, 1]$ and, more importantly, inherits from all the properties of the Hirchman Herfindahl index. In a standard IO application this index collaps into:

$$\text{HRCB}(\mathbf{p}) = \frac{\text{HHI}(\mathbf{p}) - \frac{1}{n}}{1 - \frac{1}{n}}$$

This particular version of the index HRCB appears in various literatures as a *generalized Herfindhal index* or a *normalized Herfindahl index* (see Alfano and Baraldi (2011) for instance). We have to identify, in an environment defined by propriety 1, what would be the maximal value of the HHI index.

Definition 2 (Perfect Hierarchy). *Let* $\mathbf{p}^\star$ *be a vector of points such that* $p_i = 4(n-i)$ *for* $i = 1, \ldots, n$.

Proposition 2. $\mathbf{p}^\star \in \mathscr{P}_n$*, and for any concentration index* $\mathscr{I}_n(\mathbf{p})$ *satisfying the principle of transfers we have* $\mathbf{p}^\star = \arg\max_{\mathbf{p}\in\mathscr{P}_n} \mathscr{I}_n(\mathbf{p})$ *for all* n.

First, $\mathbf{p}^\star \in \mathscr{P}_n$ because properties 1(a), 1(c) and 1(d) are obvious, $\sum_{i=1}^{n} 4(n-i) = 2n(n-1)$ (property 1(b)) and $\sum_{j=i_1}^{i_m} 4(n-j) = 4nm - 4\sum_{j=i_1}^{i_m} j = 2m(2n-m-1)$ (property 1(e)). Second,

[9] Note also that the corrections proposed by Depken (1999) or Lenten (2009) and Pawlowski et al. (2010) induce, *per se* a dependance of the maximum with respect to the number of firms/teams.

$\mathbf{p}^\star = \arg\max_{\mathbf{p}\in\mathscr{P}_n} \mathscr{I}_n(\mathbf{p})$ for all n because $\mathbf{p}^\star$ is such that it is impossible to increase the share of a team at the expense of a smaller team, then the concentration cannot increase (see Hannah and Kay, 1977).

Corollary 1. *We have* $\mathbf{p}^\star = \arg\max_{\mathbf{p}\in\mathscr{P}_n} \mathrm{HHI}(\mathbf{p})$ *for all* n *and*

$$\mathrm{HHI}(\mathbf{p}^\star) = \frac{2(2n-1)}{3n(n-1)}$$

Finally, we have:

$$\mathrm{HRCB}(\mathbf{p}^\star) = \frac{3n(n-1)}{(n+1)}(HHI - \frac{1}{n})$$

3. A probabilistic model of competitive balance

(Under construction)

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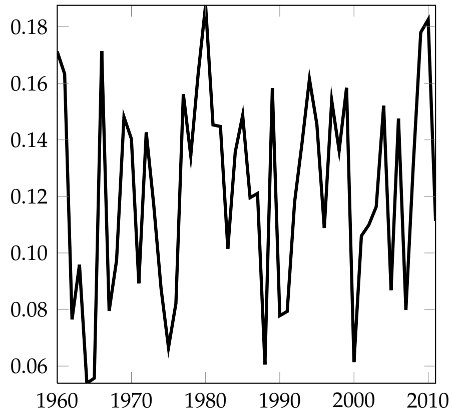
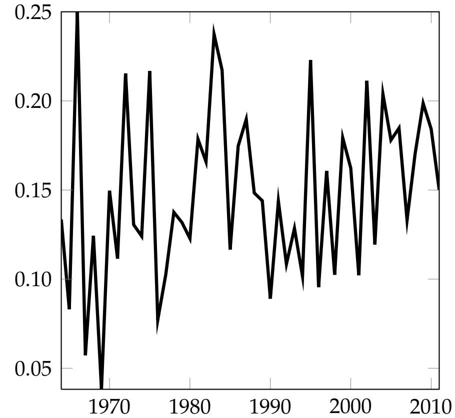
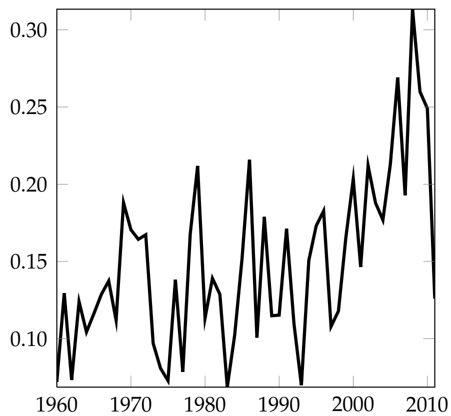


(A) France



(B) Germany



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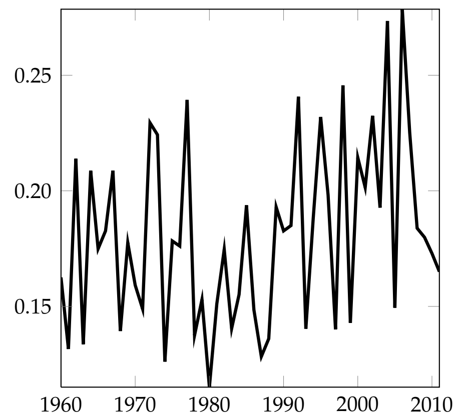
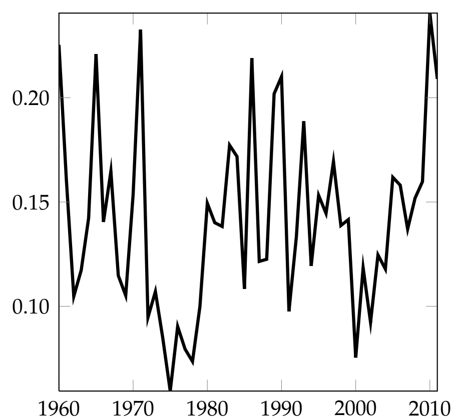


(D) Italy



(E) Spain

FIGURE 1. HRCB indices.

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