Labour Economics 27 (2014) 16–29

Contents lists available at ScienceDirect

Labour Economics

journal homepage: www.elsevier.com/locate/labeco

Turbulence, training and unemployment☆

Pascal Belan [a], Arnaud Chéron [b,*]

[a] THEMA, Université de Cergy-Pontoise, France
[b] GAINS-TEPP, Université du Maine and EDHEC Business School, France

ARTICLE INFO

Article history:
Received 12 June 2012
Received in revised form 14 January 2014
Accepted 14 January 2014
Available online 24 January 2014

JEL classification:
J24
J31

Keywords:
Training
Training subsidies
Unemployment
Matching

ABSTRACT

In this paper, we develop a matching model where firms invest in transferable human capital. Workers are endowed with heterogeneous abilities and, as a result of economic turbulence, can undergo a depreciation of their human capital during unemployment spells. Firms take inefficient training decision because they do not fully valuate the additional productivity of the workers in future jobs (poaching externality) and the additional employability after separation (unemployment externality). Higher turbulence reduces the former externality and increases the latter. It then generates some opposite forces on the gap between efficient and equilibrium training, so that it does not necessarily require higher training subsidies. The general equilibrium analysis shows that, even if the Hosios condition holds, unemployment is higher than its efficient level, which requires an additional instrument such as ability-specific employment subsidies. We lastly run some computational experiments based on the French economy to illustrate these results: optimal subsidies are found to increase with turbulence, and the total subsidy turns out to be decreasing with wages, with an efficient rate that is reduced by three from the lowest to the highest wages.

1. Introduction

Ljungqvist and Sargent (1998, 2007) emphasize that economic turbulence, featuring human capital depreciation during unemployment spell, interacts with labor market institutions to generate persistently higher unemployment in Europe than in the US. More precisely, high levels of unemployment insurance benefits and employment protection are shown to explain the unemployment gap between Europe and the US following the observed increase in the probability of skill deterioration after involuntary layoffs. Furthermore, Ljungqvist and Sargent (LS) show the plausibility of this view by assessing the role of turbulence in the context of alternative frictional labor market models. But to some extent, one caveat of this approach is to consider that the process of human capital accumulation is purely exogenous, so that it leaves no room for discussing the role of training policies. This paper aims at filling this gap by examining the efficiency and labor market policy issues of a frictional labor market model *à la* LS extended to account for endogenous training decision. It has indeed long been recognized that frictional labor markets give rise to inefficient training outcomes.

At the end of the nineties, some key contributions revisited Becker's competitive approach to human capital investments. Acemoglu (1997) first emphasizes that frictional labor market may explain the willingness of employers to bear part of the costs of general training, in contrast with the perfect labor market result. For instance, wage bargaining indeed implies that a fraction of additional productivity obtained from worker's training goes to the firm. Then, the point is that training investment in general human capital can also benefit to future employers, hence giving rise to a *poaching externality*: with some probability, an unknown party (the future employer) is getting a proportion of the training benefit when the worker is displaced.[1] Training subsidies are then required to reach optimal investment levels.[2] On another (positive) standpoint, Wasmer (2006) deals with the relative returns to specific *vs.* general human capital investments, which are found to depend both on market frictions and institutions such as employment protection. More particularly, general human capital investments are found to be more valuable in the US than in Europe, due to lower firing costs in the US. Decreuse and Granier (2013) also recently emphasized the impact of labor market institutions on the nature of educational investment, before entering the labor market. Lastly, the interaction between market imperfections and firm-sponsored training has also been documented from an empirical point of view. For instance, Picchio and van Ours (2011) showed that an in increase in labor market flexibility significantly reduces the incentives of firms to invest in training.[3]

☆ We would like to thank two anonymous referees and the co-editor of Labour Economics for their helpful comments and suggestions.
* Corresponding author at: Avenue olivier messiaen, 72035 Le Mans Cedex 9, France. Tel.: +33243833659.
E-mail address: acheron@univ-lemans.fr (A. Chéron).

[1] See also Acemoglu and Pischke (1998, 1999a,b) and Acemoglu and Shimer (1999). Stevens (1994a,b) also emphasized the role of poaching externalities for underinvestment in transferable training in a different context with imperfect competition on the labor market.

[2] On the opposite, Tripier (2011) argues that by considering intrafirm bargaining in a matching economy, efficiency of training investments typically occurs.

[3] A significant part of empirical works have also been devoted to the analysis of the wage returns to continuing vocational training; see Brunello et al. (2012) for a recent study.

0927-5371/$ – see front matter
http://dx.doi.org/10.1016/j.labeco.2014.01.001

In turn, the primarily goal of this paper is to show how economic turbulence modifies the conventional normative analysis of training. This requires to combine the turbulence explanation of unemployment as developed by LS and the literature of endogenous human capital investments in frictional labor markets. Following LS, we only deal with general human capital investment whose transferability property is assumed to be lost with some probability during workers' unemployment spell, *i.e.* specific human capital investments are not allowed for. More precisely, if we refer to the island metaphor applied to human capital investments, as developed in Wasmer (2006), firms are assumed to eventually provide workers a common technology to all islands (which is highly valuable with connected islands) but with some probability the connection fails; the greater the turbulence, the higher the probability.[4]

A first contribution of this paper is therefore to show how turbulence interacts with the poaching externality.[5] Firms can pay for some vocational training which leads to an endogenous accumulation of transferable skills, whose cost is partly shared with the workers through the wage bargaining process. If the rate of human capital obsolescence during unemployment is increased (higher economic turbulence) the probability that future employers' benefit from present training by the incumbent employer is lower. The size of the poaching externality is therefore reduced so that the gap between the equilibrium and efficient outcomes is also reduced.

Beyond that, our paper points out that taking into account of the impact of general human capital depreciation on matching probabilities can give rise to another source of externality, that we label *unemployment externality*.[6] The baseline idea is that training investments may increase the probability of leaving unemployment and contribute to raise steady-state employment, hence output. But it is obvious that firms do not internalize the consequences of their own training decisions on the unemployment level. The wedge between social and private return on training results in lower training investments than required by the first-best allocation. Furthermore, we highlight that with endogenous matching transition rates, the higher the turbulence, the higher the gap between equilibrium and efficient unemployment rates. *Ceteris paribus*, it is indeed optimal to reduce unemployment duration that exposes to a higher risk of human capital depreciation. Overall, turbulence is thus found to generate some opposite forces on the gap between efficient and equilibrium allocations.

To derive those results we develop a frictional labor market model with heterogenous workers according to observable characteristics (diplomas), where firms can invest in training that brings up-to-date knowledge to workers and raises their productivity. During unemployment spells, the worker may lose his up-to-date knowledge, but firms cannot know *ex ante* who among the unemployed underwent such a skill depreciation. Once hiring decision is taken, worker's skill is revealed and the firm chooses to pay the training cost or not. Turbulent times mean that depreciation of transferable skill during unemployment spell occurs with higher probability.

Our assumptions fit some observed patterns in firm training policies: Ok and Tergeist (2003) indeed showed, (*i*) by collecting data from 19 OECD countries that the participation rate of workers in high-skilled occupations is always higher than that of workers in low-skilled occupations, (*ii*) by running some wages regressions based on the ECHP, that the impact of training is significantly increasing with the level of workers' diploma. Accordingly, we assume that the higher the observable skill characteristic, the higher the training efficiency (see also Cunha et al. (2006) on that aspect). This implies therefore that firms concentrate training investments on high skilled workers. Additionally, accrued productivity due to training allows skilled workers to be more employable.

The theoretical analysis is begun by considering exogenous contact rates between unemployed workers and firms, assuming that high skilled workers, who are likely to be trained, have higher probability to be matched with a job than low skilled ones, who have no chance to be trained. Such a structure in contact rates is shown to be obtained in the more general benchmark matching model with endogenous contact rates. We then make a focus on the poaching and unemployment externalities and discuss the impact of turbulence on those externalities.

We lastly run some quantitative investigations of the general equilibrium model with endogenous contact rates, based on the french economy. In particular, for a calibration that satisfied the Hosios condition, the gap between equilibrium and efficient unemployment is found to rise from zero when there is no turbulence (tranquil times), to more than 2.5 points of percentage when the expected unemployment duration before experiencing human capital loss is two quarters.

Section 2 is devoted to the presentation of the benchmark matching model with turbulence and training. Section 3 characterizes the equilibrium and efficient properties of training by first assuming exogenous contact rates. Section 4 extends those former results to the endogenous matching case and runs the quantitative exercises. Section 5 concludes.

2. A matching model with turbulence and endogenous training investments

2.1. Environment and labor market flows

Time is continuous. The population of workers is a continuum of unit mass. Workers look for jobs and are randomly matched with employers looking for workers to fill vacant units of production. A productive unit is transferable job skills which can be used in present and any future occupation. For instance, workers can be trained in order to use new technologies and be aware of recent innovations in their field. These skills can be used in other jobs, but are not certified by any diploma and is therefore not observable by future employers.Moreover, during unemployment spell, the worker may lose the benefits of past firms training, and will then need a new formation to recover his up-to-date knowledge when matched with a new job. The latter assumption introduces obsolescence of human capital as a result of what Ljungqvist and Sargent (1998) have the association of one worker and one firm. Workers are heterogeneous with respect to ability a, distributed on the interval $[\underline{a}, \overline{a}]$ according to p.d.f. $f(a)$. Ability a is a general human capital index, perfectly observable by firms, due to certification through school or university diplomas.

An individual at skill level a can reach two levels of productivity, $(1+\Delta)a$ or a, with $\Delta > 0$, according to the fact that his knowledge is respectively still up-to-date or not. Additionally, firms can pay for a fixed training cost γ_F in order to provide up-to-date knowledge to the worker. Training brings additional called turbulence. It embodies the possibility of substantial human capital destruction after job loss (Jacobson et al., 1993; Farber, 2005).

For the sake of simplicity, it is assumed that workers cannot accumulate skills according to tenure and experience: either the worker has up-to-date knowledge which improves its efficiency on the job according to its ability (with an additional output equal to Δa), or his knowledge has became obsolete or depreciated which precludes any additional output.

The combination of firm training and human capital obsolescence gives rise to informational asymmetry between workers and firms during the matching process. Once the match is formed, the information on worker's skill is revealed and the negotiated wage takes account of the

[4] Wasmer (2006) argues that in an economy made of very distant islands it is better to learn the technology of the island in which one lives (specific investments), whereas the more connected the islands are, the more profitable it is to learn the common technology (general investments).

[5] Poaching externalities are generally emphasized in the context of on-the-job search. Although our paper does not allow for on-the-job search, we will also refer to poaching externality according to the fact that some employers may benefit from training of other firms.

[6] Pissarides (1992) has already stressed this kind of externality. Nevertheless, to the best of our knowledge, this issue has not yet been connected with vocational training (see Leuven (2005) for a survey).

training cost that the firm has to pay if the worker has obsolete knowledge; this results in lower wages.[7]

Training policy of a firm simply consists in determining whether new hired workers of ability a should be trained or not. In steady state, this means that some ability levels will never be trained, while other will always be trained once hired. This yields the following typology for workers. Any a-ability worker belongs to one of the three following categories: (1) type-0 individuals: unable for training; (2) type-1 individuals: able for training, but with obsolete knowledge; (3) type-2 individuals: previously trained and still highly productive.

We let $u(a)$ denote the unemployment level of a-ability workers $(e(a) = f(a) - u(a))$ and $v(a)$ the mass of job vacancies. The number of job matches taking place per unit time is given by a standard matching function $M(u(a), v(a))$.[8] The job vacancies and unemployed workers that are matched are selected randomly.

Let us denote by $u_i(a)$, the mass of unemployed a-ability workers of type i. If a-ability workers are unable for training, then they are all type-0 workers and therefore $u(a) = u_0(a)$. If, however, they are able for training, then unemployed workers are of type 1 or 2 and the unemployment level becomes $u(a) = u_1(a) + u_2(a)$. It is worthwhile to notice that type-1 and type-2 workers share the same contact rates due to informational asymmetry.

We focus on steady state equilibria where there exists a threshold ability $\widetilde{a}$ such that below, workers are of type 0, and above, workers are to type 1 or 2.[9] The process that changes the state of unemployed a-workers is Poisson. If $a \geq \widetilde{a}$, the Poisson rate is denoted by $p(a) \equiv M(\theta(a),1)$, where $\theta(a) \equiv v(a)/u(a)$ represents the tightness of the labor market for ability a. We also let $q(a)$ be the contact rate for a vacant job directed toward individuals with ability a to be matched with a worker: $q(a) \equiv p(a) / \theta(a)$. Contact rates and tightness are defined in a similar way if $a < \widetilde{a}$ and are respectively denoted by $p_0(a)$, $q_0(a)$ and $\theta_0(a)$. All jobs are assumed to separate at rate $\delta > 0$. The number of unemployed with up-to-date knowledge who experience a human capital depreciation follows a Poisson process with rate $\pi > 0$.

In steady state, for type-0 workers, inflow into unemployment $\delta(f(a) - u_0(a))$ is equal to outflow $p_0(a)u_0(a)$. This implies that, for all $a < \widetilde{a}$,

$$u_0(a) = f(a)\frac{\delta}{p_0(a) + \delta}.$$

Furthermore, for $a \geq \widetilde{a}$, $u_1(a)$ and $u_2(a)$ are derived according to the following equilibrium flow condition:

- inflow into type-1 unemployment $\pi u_2(a)$ is equal to outflow $p(a)$ $u_1(a)$.
- inflow into type-2 unemployment $\delta(f(a) - u_2(a) - u_1(a))$ is equal to outflow $(p(a) + \pi)u_2(a)$.

The resulting unemployment levels are

$$u_1(a) = f(a)\frac{\delta\pi}{(p(a)+\pi)(p(a)+\delta)} \text{ and } u_2(a) = f(a)\frac{\delta p(a)}{(p(a)+\pi)(p(a)+\delta)} \tag{1}$$

and $u(a) = u_1(a) + u_2(a) = \delta/[\delta + p(a)]$.

2.2. Steady state training and vacancy decisions of the firms

The intertemporal value of a vacant job only depends on ability a and is denoted by $V_0(a)$ for type-0 ability levels and $V(a)$ for other. Let $J_0(a)$ denote the intertemporal value of a job matched with a type-0 worker. Then, for all $a < \widetilde{a}$,

$$rV_0(a) = -c + q_0(a)(J_0(a) - V_0(a)) \tag{2}$$

$$rJ_0(a) = a - w_0(a) - \delta(J_0(a) - V_0(a)) \tag{3}$$

where $w_0(a)$ stands for the wage of type-0 workers, and r the interest rate.

Let $J_1(a)$and $J_2(a)$ denote the intertemporal values of job filled respectively with a type-1 and a type-2 worker, training in past or current job increases instantaneous production from a to $(1 + \Delta)a$. Hence, values $J_i(a)$ for $i = 1,2$, satisfy the Bellman equations

$$rJ_i(a) = (1+\Delta)a - w_i(a) - \delta(J_i(a) - V(a)) \tag{4}$$

where $w_i(a)$ is the wage for $i = 1,2$. In Eq. (4), we distinguish wages for type-1 and type-2 workers, *i.e.* we assume that a type-1 worker, once trained, cannot renegotiate his wage. This means that, at this stage, type 1 workers share the cost of training through a wage $w_1(a)$ lower than $w_2(a)$.[10]

The cost of training γ_F is payed by firms. The government can subsidize training at the time of job creation, by paying a fraction s of the training cost, so that the net training cost is $\hat{\gamma}_F \equiv \gamma_F(1-s)$.[11] The firm posting a job does not know the type of worker it will meet, but they know the aggregate composition of unemployment and therefore can calculate the probability of meeting each of the worker type: for type-1 and $u_2(a)/u(a)$ for type-2. The values of filled and vacant jobs for a-workers able to be trained $(a \geq \widetilde{a})$ satisfy the Bellman equation

$$rV(a) = -c + q(a)\left[\frac{u_1(a)}{u(a)}(J_1(a) - \hat{\gamma}_F) + \frac{u_2(a)}{u(a)}J_2(a) - V(a)\right]. \tag{5}$$

In equilibrium, free-entry implies that the rents from vacant jobs are zero: $V(a) = V_0(a) = 0$, implying that

$$J_0(a) = \frac{c}{q_0(a)} \forall a < \widetilde{a} \tag{6}$$

$$\frac{u_1(a)}{u(a)}(J_1(a) - \hat{\gamma}_F) + \frac{u_2(a)}{u(a)}J_2(a) = \frac{c}{q(a)} \forall a \geq \widetilde{a}. \tag{7}$$

Once matched, the firm immediately becomes aware of the skill of the worker, and thus identifies if the worker has experienced some skill depreciation during unemployment spell. The firm chooses to pay the training cost if the value of the filled job with training (net of training cost) is higher or equal to the value of the filled job without training. In steady-state, ability levels with training are those that satisfy $a \geq \widetilde{a}$ where $\widetilde{a}$ is defined by

$$J_1(\widetilde{a}) - \hat{\gamma}_F = J_0(\widetilde{a}). \tag{8}$$

[7] However, the newly trained worker has ex-post a clear incentive to renege on the initial wage agreement, giving rise to a hold-up problem. This point has already been emphasized in a context where firms invest in physical capital by Acemoglu and Shimer (1999). Chéron (2005) also considers the case of training costs. We discuss this issue in Section 3.6.

[8] The function M is assumed to be increasing in both its arguments, concave, and homogeneous of degree one.

[9] We discuss this point in Appendix A.1.

[10] For the moment, we leave aside hold-up problems. The only possibility for type-1 workers of getting wage $w_2(a)$is to wait for exogenous layoff and find a new employment without experiencing a human capital loss, hence becoming a type-2 worker.

[11] We abstract from the costs of subsidizing training. Either, there exists other resources for the government in the economy, or this remains to assume that all firms pay the same fixed instantaneous tax τ. In the latter case, we could add $-\tau$ on the RHS in Eqs. (3) and (4); this would not modify qualitatively the results.

2.3. Nash bargaining of wages

We consider *ex-post* bargaining, which means that it takes place after the firm observes the ability of the worker a and whether past training (if any) is obsolete or not; but it is assumed that this bargaining occurs before the firm eventually pays the training cost. Moreover, as indicated above, we consider that this negotiated wage is fixed over all the job duration (no renegotiation will occur) and deal with hold-up issues afterwards.

The present-discounted value of the expected income stream of an unemployed depends on its ability and type. Let $U_i(a)$, $i = 0,1,2$, denote this value for an unemployed of type-i. During search the worker earns some real return b that may represent home production. We assume b to be less than the lower bound of the range of abilities: $b<\underline{a}$, so that the joint return of any match without training is positive. The present-discounted value is denoted by $E_i(a)$ for a type-i worker. Those functions satisfy

$$r\mathcal{U}_0(a) = b + p_0(a)(E_0(a) - \mathcal{U}_0(a)) \tag{9}$$

$$r\mathcal{U}_1(a) = b + p(a)(E_1(a) - \mathcal{U}_1(a)) \tag{10}$$

$$r\mathcal{U}_2(a) = b + p(a)(E_2(a) - \mathcal{U}_2(a)) - \pi(\mathcal{U}_2(a) - \mathcal{U}_1(a)) \tag{11}$$

$$rE_i(a) = w_i(a) - \delta(E_i(a) - \mathcal{U}_i(a)), \quad i = 0, 2 \tag{12}$$

$$rE_1(a) = w_1(a) - \delta(E_1(a) - \mathcal{U}_2(a)). \tag{13}$$

Eqs. (11) and (13) deserve some attention. For type-2 unemployed workers, we have to take account of skill depreciation during unemployment spell, according to Poisson rate π (Eq. (11)). Moreover, since any type-1 worker benefits from training provided by his new employer, a type-1 employed becomes a type-2 unemployed in the event of job destruction (Eq. (13)).

Let β, $0 < \beta < 1$, be the bargaining power of workers. Wages are assumed to be the solutions of the following Nash-sharing rules

$$\beta J_0(a) = (1-\beta)(E_0(a) - \mathcal{U}_0(a)) \tag{14}$$

$$\beta(J_1(a) - \hat{\gamma}_F) = (1-\beta)(E_1(a) - \mathcal{U}_1(a)) \tag{15}$$

$$\beta J_2(a) = (1-\beta)(E_2(a) - \mathcal{U}_2(a)) \tag{16}$$

where the threat points of firms $V_0(a)$ and $V(a)$ have been set to zero. Since negotiation takes place immediately after the match, a new-hired type-1 worker has not been trained yet. Hence, his threat point is $\mathcal{U}_1(a)$.

Let us define $x(a) = \beta\left(\frac{r+\delta+p(a)}{r+\delta+\beta p(a)}\right)$ and $x_0(a) = \beta\left(\frac{r+\delta+p_0(a)}{r+\delta+\beta p_0(a)}\right)$. Wage equations write

$$w_0(a) = x_0(a)a + (1 - x_0(a))b \tag{17}$$

$$w_1(a) = x(a)[(1+\Delta)a - (r+\delta)\hat{\gamma}_F] + (1 - x(a))[b - \delta(\mathcal{U}_2(a) - \mathcal{U}_1(a))] \tag{18}$$

$$w_2(a) = x(a)(1+\Delta)a + (1 - x(a))[b - \pi(\mathcal{U}_2(a) - \mathcal{U}_1(a))] \tag{19}$$

where

$$\mathcal{U}_2(a) - \mathcal{U}_1(a) = \frac{\beta p(a)}{r+\pi+\beta p(a)}\hat{\gamma}_F \tag{20}$$

Property 1. *Human capital depreciation is associated with a wage loss that amounts to*

$$w_2(a) - w_1(a) = \hat{\gamma}_F \frac{\beta(r+\delta)(r+\pi+p(a))}{r+\pi+\beta p(a)} > 0, \quad \forall a \geq \tilde{a}.$$

Proof. By substituting out for $\mathcal{U}_2(a) - \mathcal{U}_1(a)$ from (20) into (18) and (19), the result follows. □

Property 1 states that holding up-to-date knowledge brings a wage premium to the worker. This is consistent with Ljungqvist and Sargent's analysis: turbulence entails substantial wage loss for workers.

Wages actually correspond to a weighted average of worker's net contribution to output and reservation wage.[12] Importantly, the reservation wages of type-1 and type-2 workers are negatively related to the unemployment gap $\mathcal{U}_2(a) - \mathcal{U}_1(a)$, or equivalently to the wage gap $w_2(a) - w_1(a)$, since Bellman equations imply:

$$\mathcal{U}_2(a) - \mathcal{U}_1(a) = \frac{p(a)(w_2(a) - w_1(a))}{(r+\delta)(r+\pi+p(a))}.$$

When matched with a job, type-1 workers expect that, in the event of job destruction at rate δ, they will enter the pool of type-2 unemployed instead of their actual type-1 position, and then can earn a higher wage in some future type-2 position.[13] This reduces type-1 reservation wages at the time of bargaining. Furthermore, type-2 workers expect that if the bargaining process fails they face a risk of human capital depreciation, occurring at rate π, which accounts for a loss $\mathcal{U}_2(a) - \mathcal{U}_1(a)$. This lowers type-2 reservation wages.

2.4. The discontinuity of the steady-state arrival rates

From Eqs. (33)–(38) in Appendix A.1, the characterization of the steady-state labor market equilibrium can be stated as follows:

$$\frac{c}{q_0(a)} = \frac{(1-\beta)(a-b)}{r+\delta+\beta p_0(a)} \tag{21}$$

$$\frac{c}{q(a)} = \frac{(1-\beta)((1+\Delta)a-b)}{r+\delta+\beta p(a)} - \frac{(1-\beta)\hat{\gamma}_F}{r+\delta+\beta p(a)}\left(\frac{\pi}{\pi+p(a)}\right)\left[(r+\delta) - \frac{(\delta+p(a))\beta p(a)}{r+\pi+\beta p(a)}\right] \tag{22}$$

$$\Delta\tilde{a} = \hat{\gamma}_F\left(r + \frac{(r+\pi)\delta}{r+\pi+\beta p(\tilde{a})}\right) + (\tilde{a} - b)\beta\left(\frac{p(\tilde{a}) - p_0(\tilde{a})}{r+\delta+\beta p_0(\tilde{a})}\right). \tag{23}$$

A general discussion of the existence and uniqueness of this equilibrium is provided in Appendix A.1; Section 4.2.2 will also look at some numerical experiments. At this stage, our point is to examine the occurrence of a discontinuity point of the arrival contact rates around the threshold ability $\tilde{a}$.

Property 2. *Let us assume that a steady state equilibrium exists and is unique. If (8) is satisfied for some ability level a, then $p(a) > p_0(a)$.*

Proof. From the Bellman Eqs. (3)–(4) and the wage Eqs. (17)–(20) one deduces that (8) implies

$$J_2(a) > J_1(a) - \hat{\gamma}_F \geq J_0(a).$$

Then, the free-entry Eqs. (6) and (7) imply

$$\frac{c}{q(a)} > \frac{c}{q_0(a)}$$

which leads to the result. □

This discontinuity point results from imperfect information of firms about workers' productivity. Since firms cannot *ex ante* observe whether an unemployed worker has up-to-date knowledge or not, they cannot discriminate between type-1 and type-2 workers before entering

[12] The latter refers to the wage earnings when assuming $\beta \to 0$ (which means $x = x_0 = 0$).

[13] This can account for a hold-up issue, which leads to additional source of inefficiencies, as discussed in Section 3.6.

the matching process. At the ability level $\tilde{a}$, a worker either have up-to-date knowledge and productivity $(1+\Delta)\tilde{a}$, or he will be trained in his future position at cost $\hat{\gamma}_F$ in order to reach this same level of productivity. By contrast, for ability levels a just below $\tilde{a}$, worker's productivity is a and firm pays no training cost. Then, by posting a vacant position directed toward workers with ability higher or equal to $\tilde{a}$, the firm expects that it may meet a type-2 worker whose up-to-date knowledge allows to reach a higher productivity without any additional cost. This implies that, everything else being equal, more firms would like to enter.

3. Turbulence and training: The partial equilibrium case

To allow easier understanding of our results, let us first assume exogenous contact rates with a discontinuity at $a=\tilde{a}$:

$$p(a)=\begin{cases} p_0 \text{ if } a<\tilde{a} \\ p \text{ for type-1 and type-2 individuals with } a\geq\tilde{a}\end{cases}$$

with $p_0 \leq p$.

3.1. The impact of turbulence on equilibrium training

Our goal here is to show how turbulence affects this threshold. We restrain our analysis to economies where workers with the lowest ability $\underline{a}$ are not trained in equilibrium, that is $\tilde{a}>\underline{a}$. Proposition 1 gives a necessary and sufficient condition for the existence of such an ability threshold $\tilde{a}$.

Proposition 1. *With Nash-bargaining wage setting, the equilibrium ability threshold $\tilde{a}$ is larger than $\underline{a}$ and characterized by:*

$$\Delta\tilde{a}=\hat{\gamma}_F\left(r+\frac{(r+\pi)\delta}{r+\pi+\beta p}\right)+(\tilde{a}-b)\left(\frac{x-x_0}{1-x}\right) \tag{24}$$

if and only if the following condition is satisfied

$$(1-x)(1+\Delta)>(1-x_0) \quad \text{and} \quad \gamma_F\left(r+\frac{(r+\pi)\delta}{r+\pi+p}\right)>\Delta\underline{a}. \tag{25}$$

Proof. Combining Eqs. (8) with (17), (18) and (20) leads to Eq. (24). Condition (25) then implies that $\tilde{a}>\underline{a}$.□

The first inequality in condition (25) is equivalent to $(1-x)(1+\Delta)a>(1-x_0)a$ which means that, once the training cost has been paid, the instantaneous profit value of training is positive and increases with a. Since $p\geq p_0$, the "effective" worker's shares of instantaneous surplus are such that $x\geq x_0$. Therefore, Δ has to be large enough to imply that the instantaneous profit of the firm increases with training. Since the training cost is fixed, high-ability workers are more likely to be trained than low-ability ones. The second inequality in (25) implies that workers with the lowest ability $\underline{a}$ are never trained.

Overall unemployment

$$u=\int_{\underline{a}}^{\tilde{a}}\frac{\delta f(a)}{p_0+\delta}da+\int_{\tilde{a}}^{\bar{a}}\frac{\delta f(a)}{p+\delta}da \tag{26}$$

is then negatively related to the share of workers who are eligible for training as soon as $p>p_0$. The effect of turbulence on unemployment therefore depends on the relationship between $\tilde{a}$ and π.

Property 3. *Under condition (25), a higher turbulence reduces the share of trained workers and, for $p>p_0$, raises the unemployment rate.*

Proof. From Proposition 1, it comes that

$$\tilde{a}=\frac{\hat{\gamma}_F\left(r+\frac{(r+\pi)\delta}{r+\pi+\beta p}\right)-b\left(\frac{x-x_0}{1-x}\right)}{\Delta-\frac{x-x_0}{1-x}}.$$

The first inequality in condition (25) implies that the denominator is positive. The threshold $\tilde{a}$ is then unambiguously increasing with respect to π. Moreover, Eq. (26) implies that, if $p>p_0$, overall unemployment is positively related to the proportion of type-0 workers, $F(\tilde{a})$. This concludes the proof. □

To understand Property 3, it is worth emphasizing that the equilibrium training rule highly depends on the expected unemployment surplus related to training $\mathcal{U}_2(a)-\mathcal{U}_1(a)$. Formally, the derivation of Proposition 1 indeed relies on the fact that the equilibrium ability threshold solves:

$$\Delta\tilde{a}=(r+\delta)\hat{\gamma}_F-\delta(\mathcal{U}_2(a)-\mathcal{U}_1(a))+(\tilde{a}-b)\left(\frac{x-x_0}{1-x}\right) \tag{27}$$

where the unemployment gain $\mathcal{U}_2(a)-\mathcal{U}_1(a)$ is defined by (20). Higher turbulence reduces the unemployment gain, because workers expect to switch more quickly from type-2 to type-1 unemployment. Therefore, turbulence increases the reservation wage of type-1 workers and raises the ability threshold $\tilde{a}$. Otherwise stated, turbulence discourages firms to train by increasing threat points of type-1 workers.[14] Then, the fraction of untrained workers $F(\tilde{a})$ who face a lower probability of exiting unemployment $(p_0<p)$ increases with π. From Eq. (26), this results in a rise in the overall unemployment rate.

3.2. Efficient training policy

In order to determine the efficient training decision, we consider the problem of a social planner that maximizes the expected sum of social output subject to the same matching and informational constraints as the decentralized economy. The results summarized in the following proposition are derived in Appendix A.2.

Proposition 2. *Under condition (25), the efficient training ability threshold a^* is larger than $\underline{a}$. It satisfies and is increasing with turbulence.*

$$\begin{aligned}\Delta a^* &= \left(r+\frac{(r+\pi)\delta}{r+\pi+p}\right)\gamma_F-\left(a^*-b\right)\left(\frac{r+\delta}{r+\delta+p_0}\right)\frac{p-p_0}{p} \\ &<\Delta\tilde{a}=\left(r+\frac{(r+\pi)\delta}{r+\pi+\beta p}\right)\hat{\gamma}_F+(\tilde{a}-b)\frac{\beta(p-p_0)}{r+\delta+\beta p_0}.\end{aligned} \tag{28}$$

Proof. Eq. (39) in Appendix A and condition (25) implies that the efficient threshold a^* above which workers should be trained is characterized by (28) and is larger than $\underline{a}$. The rest of the proposition follows immediately. □

Efficient training threshold is always lower than the equilibrium one. Even if efficiency does not necessarily imply that all workers should be trained once hired, the mass of workers who benefit from training in equilibrium is too low. Then, turbulence has the same effect on the optimal ability threshold as on the equilibrium one. From the social planner's point of view, it is indeed less worthwhile to train workers since turbulence increases the probability of losing training investment costs. Hence, low-ability workers who could be trained in the context of low turbulence, are no longer trained if the turbulence is high.

In order to disentangle the externalities that make equilibrium inefficient, we focus on the two following extreme configurations: (i) $p_0=p$ and $\beta<1$ and (ii) $p_0<p$ and $\beta\to 1$. The well-known *poaching externality* appears as soon as the worker does not benefit from the full return of the training investment, *i.e.* as soon as β is lower than one. In the following, we stress that there is also another externality that we call *steady-state unemployment externality* that appears as soon as the contact rates differ between workers who will never benefit training and

[14] It should be emphasized that this result is consistent with an average wage that decreases with turbulence. Indeed, the share of untrained workers who earn the lowest wage $w_0(a)$ is increasing with π.

those who will be retrained if necessary. If workers below some ability threshold are never trained, they face higher unemployment duration. Then the higher is the threshold, the higher overall unemployment. Assuming $p_0 = p$ (case (i)) allows to eliminate the unemployment externality and focus on the poaching externality. On the contrary, case (ii) assumes β close to one which eliminates the poaching externality.

3.3. The poaching externality

Consider $p_0 = p$ in Eq. (28). The difference between efficient and equilibrium thresholds only comes from the fact that β is lower than one. In line with Acemoglu (1997), imperfect labor market implies that firms do not internalize the entire social gain of their own training decision. Training does not only increase productivity of the worker in the current firm but also increases productivity in future jobs if his human capital does not depreciate during unemployment spells. This point leads us to refer to the *poaching externality* although our model does not allow for on-the-job search. Unlike firms, workers may internalize this value of training (higher expected wages in other firms). Nevertheless, since $\beta < 1$, they only get a fraction of the additional productivity related to training when they move to another job. Thus, the relative unemployed value of having up-to-date knowledge $(\mathcal{U}_2(a) - \mathcal{U}_1(a))$ is not high enough. Consequently, the reservation wage of type-1 workers is too high and the number of people kept out of training is higher than the efficient level. Of course, as β goes to unity, workers tend to capture the total gain related to training. The gap between equilibrium and efficient thresholds would then tend to zero (when both $p_0 = p$ and $\beta \to 1$).

3.4. The unemployment externality

We now abstract from the poaching externality by assuming that β is arbitrarily close to unity and go back to the case $p_0 < p$. From the point of view of the social planner, Eq. (28) shows that a lower contact rate for type-0 workers reduces the efficient ability threshold a^*, so that it increases the fraction of workers who should be trained (Fig. 1). Keeping a worker of ability a out of training leads indeed to a social output loss. Firms do not valuate this loss, whereas the planner does. Then, obviously, the size of the externality is greater when the difference between market and home productions is large, *i.e.* the social cost of a rise in unemployment increases with $a^* - b$. In addition, assuming $p - p_0$ contributes to increase the equilibrium ability threshold $\tilde{a}$ – in the context of Nash bargaining – by raising the effective share of instantaneous profit that goes to employees (x instead of x_0), hence wages. Accordingly, from both arguments, firms under invest in training. Then, too many workers face a low probability to exit unemployment. Therefore, the equilibrium unemployment rate is higher than the efficient one.

3.5. Turbulence and the optimal training subsidies

Differences between the equilibrium and the optimum come from the gap between the efficient ability threshold and its equilibrium value. We now address the issue of the optimal policy. Does a subsidy of the training cost allow to decentralize the optimum? Moreover, how should it evolve according to the level of turbulence? Otherwise stated, should the subsidy rate increase or decrease when the probability of losing human capital during unemployment spell raises? Our point is that the relative importance of the two externalities presented above, either "poaching" or "steady-state unemployment", determines the effect of turbulence on the optimal level of subsidy.

Proposition 3. *Under condition (25), the optimal subsidy rate of training is:*

$$s^* = \left[\frac{(r+\pi)\delta}{(r+\pi)(r+\delta)+r\beta p}\right]\left[\frac{(1-\beta)p}{r+\pi+p}\right] + \left[\frac{r+\frac{(r+\pi)\delta}{r+\pi+p}-\frac{\Delta b}{\gamma_F}}{r+\frac{(r+\pi)\delta}{r+\pi+\beta p}}\right] \times \left[\frac{\frac{r+\delta}{r+\delta+p_0}\frac{(p-p_0)}{p}+\frac{\beta(p-p_0)}{r+\delta+\beta p_0}}{\frac{r+\delta}{r+\delta+p_0}\frac{p-p_0}{p}+\Delta}\right].$$

Proof. Using Propositions 1 and 2 with $\hat{\gamma}_F = (1-s^\star)\gamma_F$, s^* is the value of the subsidy rate such that $\tilde{a} = a^\star$. □

To analyze the effect of turbulence, in a preliminary step, we take the two specifications considered above, $\{\beta < 1, p_0 = p\}$ and $\{\beta = 1, p_0 < p\}$, since they allow to focus separately on each externality at work. Intermediate cases will then be considered at the end of this section.

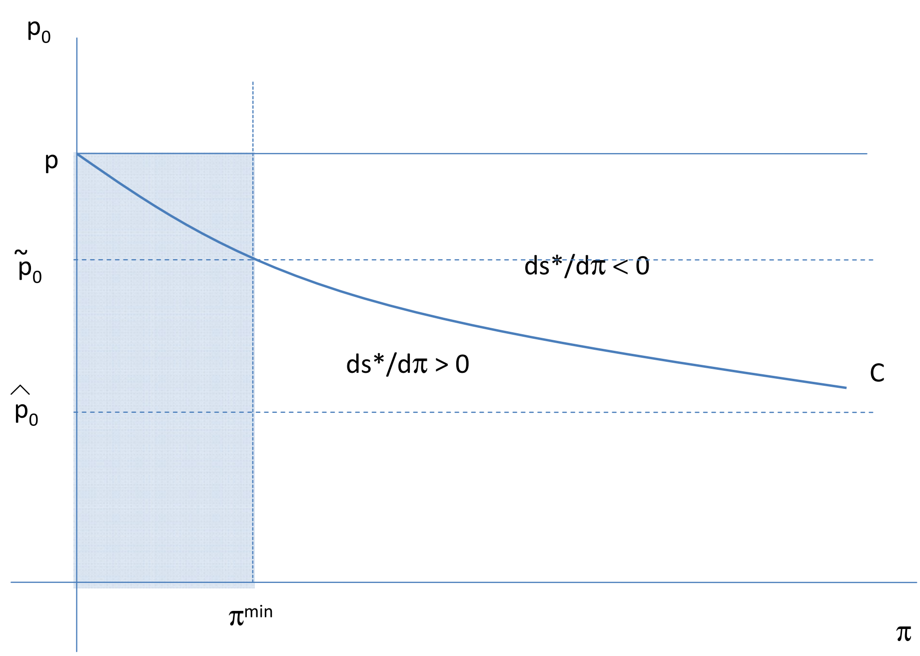

Fig. 1. Turbulence parameter (π) and the optimal subsidy rate (s^*), according to the contact rate of type-0 workers (p_0).

Corollary 1. *Consider $p_0 = p$ and $r = 0$. The optimal rate of training subsidy is:*

$$s^\star = \frac{(1-\beta)p}{\pi + p} \tag{29}$$

and is decreasing with the Poisson rate of human capital depreciation π.

For $\beta < 1$, it is efficient to subsidize the training cost due to the poaching externality. Corollary 1 states that the optimal subsidy rate should decrease with turbulence. Both equilibrium and efficient ability thresholds are increasing with π but, the higher the turbulence, the lower the size of the poaching externality. Indeed, $\tilde{a}$ tends to α^* as the Poisson rate π goes to infinity. When the turbulence is high, a firm would less likely benefit from training decision of other firms; the social return to training investment converges to its private return. Ultimately, if the rate π goes to infinity, the probability of experiencing a human capital depreciation tends to unity. Then, training investment collapses to job-specific human capital, so that both equilibrium training policy and unemployment rate are optimal and no subsidy is required. From that point of view, we need less training subsidies in the context of high turbulence.[15]

Let us now consider the specification which eliminates the poaching externality: $p_0 < p$ and $\beta \to 1$.

Corollary 2. *Consider $p_0 < p$, $\beta \to 1$ and $r = 0$. The optimal subsidy rate of training is:*

$$s^* = \frac{\delta + p}{\delta + \Delta\frac{p(\delta+p_0)}{p-p_0}}\left(1 - \frac{\Delta b}{\delta\gamma_F}\frac{\pi + p}{\pi}\right)$$

and is increasing with the Poisson rate of human capital depreciation π.

Corollary 2 only deals with steady-state unemployment externality. It turns out that a higher rate of human capital depreciation leads to implement a higher training subsidy rate in order to restore efficiency. Both efficient and equilibrium training ability thresholds are still increasing with economic turbulence, but the gap between them also increases with respect to π.[16] Indeed, the higher the rate π, the higher the output loss $(\tilde{a} - b)$ during unemployment spells of type-0 workers. Unlike the central planner, firms do not internalize the social incidence of an increase in ability threshold. Therefore, the higher the turbulence, the larger the gap between equilibrium and efficient thresholds. Consequently, the training subsidy rate increases with π.

As a result, the overall impact of turbulence on the subsidy rate of training cost is ambiguous. In response to higher turbulence, the increase in unemployment externality introduces an opposite force to the reduction of inefficiencies related to the poaching externality. It is thus unclear whether decentralization of the efficient threshold leads to greater training subsidies in the context of higher economic turbulence or not.

Let us now consider the intermediate case where both externalities are simultaneously at work. Fig. 1 reports the sign of the marginal impact of an increase in turbulence on the optimal subsidy rate of training, according to the Poisson rate of human capital depreciation and the contact rate for type-0 workers, (π, p_0). It shows that three regimes typically emerge, according to the value of p_0:[17]

- If $p_0 \le \hat{p}_0$, then the unemployment externality is so high that an increase in turbulence always requires to increase the subsidy rate of training.
- If $p_0 \ge \tilde{p}_0$, then the unemployment externality is so low that an increase in turbulence always requires to reduce the subsidy rate of training.
- For intermediate values $p_0 \in (\hat{p}_0, \tilde{p}_0)$, there exists a threshold initial value for π (defined by C) above which an increase in turbulence requires a cut in the subsidy rate of training.

3.6. The impact of hold-up

With Nash bargaining wage setting process, it is obvious that workers have *ex-post* a clear incentive to renege on the initial wage agreement. Type-1 worker can become type-2 worker (with higher wage) only if he gets unemployed and finds a new job quickly enough before losing his up-to-date knowledge. Without enforceable contractual arrangement of the type developed by Malcomson (1997), a hold-up problem may then arise. Until now, we have considered an extreme assumption – more favorable to see emerging efficient outcomes – which has allowed us to make a focus on the poaching externality and the additional unemployment externality.

We propose now to deal with the other polar case where type-1 workers are able to bargain initially the same earnings as type-2 workers.[18] Then, since we assume homogenous contact rate p for type-1 and type-2 unemployed workers, it comes that $U_1 = U_2$. Denoting by $w^h(\alpha)$ the wage for workers with ability $a \ge \tilde{a}$, it is now straightforward to see that:

$$w^h(a) = x(1+\Delta)a + (1-x)b.$$

Proposition 4. *Consider $r = 0$. In the context of unenforceable wage contracts, under condition (25), the equilibrium training ability threshold α^h is larger than b, it satisfies*

$$\Delta a^h = \hat{\gamma}_F\left(\frac{\delta + \beta p}{1-\beta}\right) + \left(a^h - b\right)\beta\left(\frac{p - p_0}{\delta + \beta p_0}\right) > \Delta\tilde{a}$$

and it does not depend on turbulence.

In the context of contractual incompleteness, each type-1 worker has *ex-post* incentives to renege on the wage contract and claims for type-2 wage level. Accordingly, firms face an additional wage cost that moves upward the training threshold. Moreover, since wages are (re) negotiated after training, turbulence has no longer impact on wages and equilibrium training rules. It also appears that the sign of the relationship between the optimal subsidy rate of training and turbulence is now clear cut even though $p > p_0$.

Proposition 5. Consider $r = 0$ and $p_0 = 0$. Under condition (25), the optimal rate of training subsidy is

$$s^h = 1 - \frac{\Delta}{1+\Delta}(1-\beta)\left[\frac{\pi}{\pi + p}\left(\frac{\delta}{\delta + \beta p}\right)\left(1 - \frac{\beta p}{\Delta\delta}\right) + \frac{b}{\delta\gamma_F}\right]$$

and it is decreasing with π.

---

[15] To avoid useless complexity, we have focused on the case $r = 0$. Nevertheless, with $r > 0$, the optimal subsidy rate is increasing with turbulence for low values of π, and becomes decreasing above the threshold $\left(\frac{r\delta}{r+\delta}\right)^{1/2} p - r$.

[16] We have indeed $\tilde{a} - a^\star = \left[1 + \frac{p}{p-p_0}\frac{\delta + p_0}{\delta}\Delta\right]^{-1}\left(1 + \frac{p}{\delta}\right)(\tilde{a} - b)$.

[17] On Fig. 1, π^{min} corresponds to the lower bound of the Poisson rate π that satisfies the second inequality in condition (25) for $s = 0$ and $r = 0$. Considering π larger than π^{min} means that $\tilde{a}$ and $a^\star$ are both greater than b. The curve C is the set of values of π and p_0 such that $\frac{ds}{d\pi} = 0$. C is decreasing, passes through the point $(0,p)$ with an asymptote $p_0 = \hat{p}_0$ for $\pi \to +\infty$. Finally, $\tilde{p}_0$ is the value of p_0 where the curve C intersects the vertical line $\pi = \pi^{min}$.

[18] Moreover, notice that there is no shock that could provide some grounds for two-tier wage contracts (as in Mortensen–Pissarides). It could also be possible to introduce an exogenous probability of wage renegotiation where the cost of training is shared by the two parties only before this renegotiation. However, without any other constraints or distortions, the labor market outcomes are unchanged with respect to the case where the wage is rigid and the costs of training shared during all the job duration. It comes indeed that the higher the probability of renegotiation is, the lower the wage during the initial period (lower than our rigid wage) because firms expect additional wage costs related to renegotiation (proof available upon request).

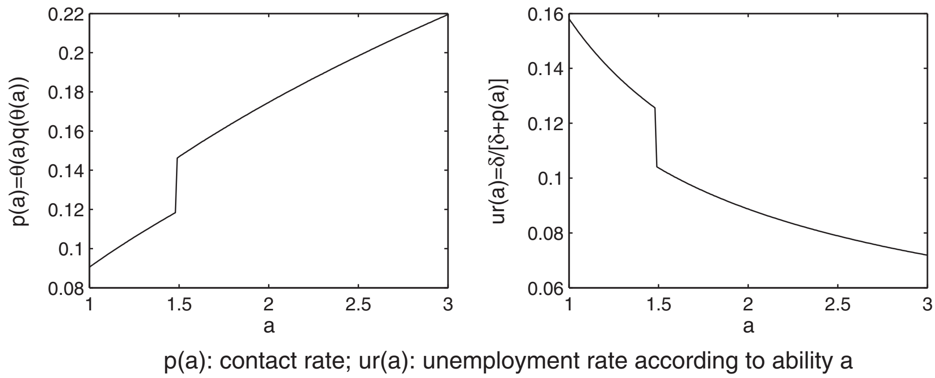

Fig. 2. Equilibrium with Nash-bargaining: Benchmark case. $p(a)$: contact rate; $ur(a)$: unemployment rate according to ability a.

An increase in the Poisson rate of human capital depreciation π unambiguously implies a decrease in the optimal subsidy rate of training, and despite we are looking at a parameter configuration where the steady-state unemployment externality is at its highest value (with $p_0 = 0$). The intuition behind this result is as follows. On the one hand, the equilibrium ability threshold α^h is no longer related to the Poisson rate of human capital loss. This occurs because, in the context of hold-up, workers do not expect any wage loss in the event of human capital depreciation. On the other hand, the efficient ability threshold still increases with turbulence, because it is less worthwhile from the planner's point of view to pay the training costs which are more likely to become unproductive. As a consequence, the gap between efficiency and equilibrium thresholds unambiguously decreases when the Poisson rate of human capital depreciation raises. The higher the turbulence, the lower is the optimal subsidy rate of training.

4. Turbulence and (in)efficiency with endogenous matching

We now go back to the initial framework with endogenous arrival rates and analyze how both externalities (poaching and steady-state unemployment) interact with the equilibrium arrival rates.

4.1. Efficient allocation and optimal policy with endogenous matching

A general presentation of the social planner program is provided in Appendix A.2.2. Hereafter, we consider $r \to 0$ to show optimal conditions and discuss the efficiency properties. Let us define $\psi \equiv 1-\frac{\theta p'(\theta(a))}{p(\theta(a))}$.

Using stars in superscript to denote endogenous variables at their optimal values, the efficient allocation is characterized by:

$$\frac{c}{q_0^\star(a)} = \frac{(1-\psi)(a-b)}{\delta+\psi p_0^\star(a)}, \forall a<a^\star \tag{30}$$

$$\frac{c\delta}{q(\theta^\star(a))} = \frac{(1-\psi)((1+\Delta)a-b)}{\delta+\psi p^\star(a)} - \frac{\gamma_F}{\delta+\psi p^\star(a)}\left(\frac{\pi}{\pi+p^\star(a)}\right)\left[\delta-\frac{\delta+p^\star(a)}{\pi+p^\star(a)}p^\star(a)\right], \forall a\geq a^\star \tag{31}$$

$$\Delta a^\star = \gamma_F\frac{\delta\pi}{\pi+p^\star(a^\star)} - \frac{(a^\star-b)\delta}{\delta+p_0^\star(a^\star)}\left(\frac{p^\star(a^\star)-p_0^\star(a^\star)}{p^\star(a^\star)}\right) - \frac{c\delta}{q^\star(a)}\left(\frac{\delta+p^\star(a)p_0^\star(a)q^\star(a)}{\delta+p_0^\star(a)p^\star(a)q_0^\star(a)}-1\right). \tag{32}$$

Property 4. *The Hosios condition $\psi = \beta$ does not achieve efficiency.*

Proof. Assuming $\psi = \beta$, $r = 0$ and $\hat{\gamma}_F = \gamma_F$ $(s = 0)$ in Eqs. (21)–(23), and compare with (30)–(32) leads to the result. □

First, it is obvious that, due to the poaching and steady-state unemployment externalities, the equilibrium ability threshold does not correspond to its efficient counterpart $(\tilde{a}\neq a^\star)$; see the discussion in Section 3.2, which is here generalized to endogenous contact rates, hence $c > 0$.

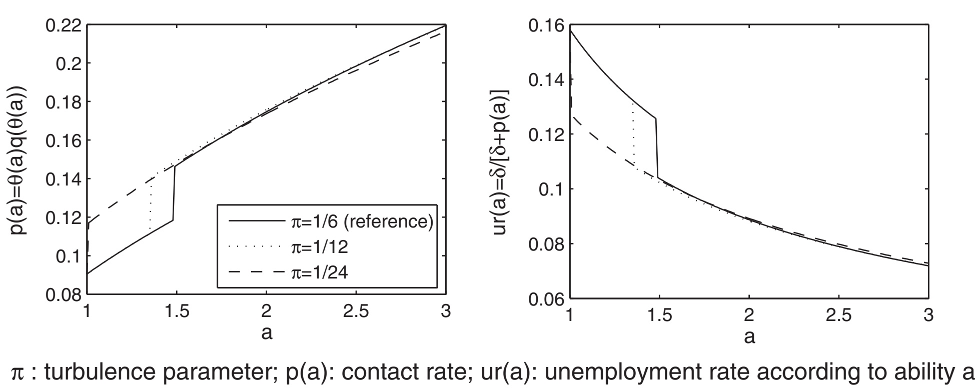

Fig. 3. Equilibrium with Nash-bargaining: The impact of turbulence. π: turbulence parameter; $p(a)$: contact rate; $ur(a)$: unemployment rate according to ability a.

But importantly, even if training was optimal $\left(\tilde{a} = a^{\star}\right)$, efficiency would still not be achieved. Indeed, with positive investment costs γ_F, Eqs. (22) and (31) are still different. This is due to the fact that the valuation of the gap $U_2 - U_1$, which is related to training costs paid through wage cuts by type-1 unemployed when they come back to employment, remains lower than the corresponding valuation by the planner. Accordingly, reservation wages of type-1 and type-2 workers are too high and this pushes down labor market tightness for workers with ability $a \geq \tilde{a}$.

Two policy tools are therefore required to restore equilibrium efficiency. Even though the Hosios condition would be satisfied, the training subsidy at rate s should be completed by another instrument.

4.2. Quantitative illustration

We now go back to the properties of the equilibrium model with endogenous matching and implement some numerical experiments. On the one hand, we want to illustrate to what extent turbulence affects equilibrium training and unemployment and, on the other hand, we examine how the gap between the equilibrium and the efficient allocations is sensitive to turbulence. This quantitative section is therefore divided into three steps. We first deal with the impact of the training cost and turbulence parameter on some key labor market outcomes (unemployment rate, share of trained workers, labor market tightness) by considering the conventional Nash bargaining. Secondly, we explore the quantitative incidence of hold-up. Lastly, we compare those equilibrium outcomes to efficient counterparts. As a preliminary step, we start by presenting the benchmark case.

4.2.1. Benchmark case

We calibrate the model on a monthly basis. Key parameters are chosen to fit stylized facts that feature the French labor market, by referring to the recent paper by Hairault et al. (2012) which provides new evidence on labor market flows in France. The design of our calibration strategy is such that it remains only one free parameter, γ_F, that we let vary to show the robustness and sensitivity of our quantitative results.

We first need to specify some functional forms. As regards to the matching process, we assume a basic Cobb–Douglas function consistent with $q(\theta) = \Gamma\theta^{\alpha-1}$ and $p(\theta) = \Gamma\theta^{\alpha}$ with $\alpha \in (0,1)$. Secondly, the heterogeneity of abilities satisfies the following Pareto distribution function $F(a) = \frac{3}{2}(1-\frac{1}{a})$ for $\alpha \in [1,3]$. This implies that the highest wage is 4 times greater than the lowest wage in the economy, suggesting that we only leave aside the top-5% of the wage distribution.

Then, a first set of parameters is fixed in accordance with the existing literature. In a fairly standard way, we set $b = 0.2$ and consider $\alpha = \beta = 0.5$. As stated earlier, the latter condition, which reflects the well-known Hosios condition, no longer guarantees efficiency in our context. A second subset of parameters is calibrated to fit stylized facts of the labor market in France. Namely, we set $c = 2.3$ and $\Gamma = 0.18$, to be consistent with an average monthly job finding rate of 0.14, and an average labor market tightness of 0.65. The monthly job destruction rate is $\delta = 0.017$.

Next, we follow Den Haan et al. (2005) (dHHR) and Ljungqvist and Sargent (2004) (LS), who focused on the impact of turbulence (in contexts without any endogenous training decisions) to calibrate $\Delta = 0.25$, consistent with an increase of 25% of the average productivity between high and low productivity workers (as in dHHR), and $\pi = 1/6$, implying that, for unemployed workers, the expected probability during a quarter to fall from type-2 to type-1 is 50% Ljungqvist and Sargent (2004). Moving from present turbulent times to more tranquil times means a fall of π; we will consider alternative values lower than 1/6.

4.2.2. Equilibrium with Nash bargaining

Fig. 2 and Table 1 show some key outcomes on the labor market in the benchmark case and under the assumption of Nash wage bargaining. We take $\pi = 1/6$ as a reference for turbulent times, and we let vary γ_F to show the sensitivity of the results, $\gamma_F = 12$ being a reference value consistent with a training cost that represents 20% of the lowest productivity $\underline{a}$ over the average match duration, $1/\delta(\frac{\underline{a}}{\delta} = \frac{1}{0.017} \simeq 59)$.[19] With this reference value, it comes that $\tilde{a} = 1.5$ and half of the workers are never trained ($F(\tilde{a}) = 0.5$ in Table 1), which is consistent with empirical evidence for France.[20] Around this threshold ability value, we also observe a discontinuity of the job finding probability, consistent with an increase of the expected unemployment duration from 6.6 months when $a = \tilde{a} = 1.5$ to more than 8.5 months for values of a just below $\tilde{a}$.

Then, in line with our comparative static analysis, the main results are the following (see Table 1 and Fig. 3):

- The higher the training cost, the lower the share of trained workers and the higher the overall unemployment rate. Moreover, as could be expected, the share of trained workers is highly sensitive to the value of γ_F.
- Higher turbulence results in a lower share of trained workers and a higher overall unemployment rate. Indeed, higher turbulence lowers the relative value of being trained, and leads to higher wages for type-1 workers.
- In the benchmark case ($\gamma_F = 12$), the rise in turbulence (from tranquil times with $\pi = 1/24$ to turbulent times with $\pi = 1/6$) leads to an increase by 1.2 percentage point of the unemployment rate and excludes almost half of the workers from the training process by firms (indeed, $F(\tilde{a})$ goes from 3% to 50% when π goes from 1/24 to 1/6).

4.2.3. Equilibrium with hold-up

Let us now allow for the hold-up problem to arise in the wage determination process. In the benchmark case (with $\gamma_F = 12$), none of the workers would then be trained in equilibrium, whereas, without hold up, half of the workers are trained (see Fig. 4 and Table 2). The possibility of hold-up introduces an upward pressure on wages, which makes training investment unprofitable even for the highest ability workers and leads to higher unemployment.

Moreover, the training cost γ_F should be higher than 2.5 for getting a share of trained workers lower than one. Table 2 reports the results for $\gamma_F = 3$ and several values of π. The sensitivity of the unemployment rate is found to be quite lower than in the case of Nash bargaining (because wages are no longer affected by π): about 0.7 point of percentage (instead of 1.2) when going from $\pi = 1/24$ to $\pi = 1/6$. This unemployment impact indeed only reflects a composition effect which reduces the number of vacancies when there is a rise in turbulence: a higher π raises $u_1(a)/u(a)$, which means that when posting a vacancy, the probability to support the training cost is higher for the firms; this pushes vacancies downward and unemployment upward. By contrast, without hold-up, there was an additional negative effect of turbulence on unemployment: a rise in turbulence increases wages, due to a higher gap between unemployment values $U_2 - U_1$. This wage adjustment amplifies the composition effect and therefore accounts for a higher increase in unemployment.

4.2.4. Efficiency

We examine the efficiency properties of our matching model from a quantitative standpoint. The planner internalizes both the poaching and the unemployment externalities. In the benchmark case, those externalities are so large that all the workers are trained; as plotted on Fig. 5, the optimal job finding rate is increasing and continuous with respect to ability. Actually, $F(a^*)$ turns out to be positive only if $\gamma_F \geq 24$, that is if

---

[19] $\gamma_F = 12$ represents also approximatively seven months of the computed average productivity of employed workers, that is $\int_1^{a} a(f(a) - u(a))da + \int_{\tilde{a}}^{3}(1+\Delta)a(f(a) - u(a))da \simeq 1.7$ with $\tilde{a} = 1.5$.

[20] In France, we do observe a strong selection effect in continuous training program, according to diploma and age (see for instance DARES (2012)). In particular, the participation rate to continuous vocational training for low-skilled workers is less than half that of the high-skilled. Furthermore, the overall participation rate on firm-sponsored CVT for the 20–40 years old workers is about 50%.

Table 1
Numerical results: Equilibrium with Nash bargaining.

	$\pi = 1/6$ (ref.)			$\gamma_F = 1/12$ (ref.)			$\gamma_F = 1/15$		
	$\gamma_F = 9$	12 (ref.)	15	$\pi = 1/6$ (ref.)	1/12	1/24	$\pi = 1/6$	1/12	1/24
$F(\tilde{a})$ in %	0	**50**	82	**50**	40.5	3	82	70.2	38
u in %	10.5	**11.5**	12.1	**11.5**	11.2	10.3	12.1	11.9	11.2

π: turbulence parameter; γ_F: training costs; $F(\tilde{a})$: share of workers not trained; u: unemployment rate

the training cost is higher than approximately one year of the average productivity. Fig. 5 then shows arrival rate and unemployment rate at the optimum and at the equilibrium without hold-up for the reference parametrization with $\gamma_F = 12$.

Keeping the reference value $\gamma_F = 12$ for the training cost, Table 3 gives the share of untrained workers, the unemployment rate and welfare at the equilibrium without hold-up and at the optimum for several values of the turbulence parameter π. If π goes to zero, both poaching and unemployment externalities vanish: all workers are trained at the optimum and at the equilibrium; they never lose their skills. Since by assumption the Hosios condition is satisfied, equilibrium and optimal levels of welfare are equal. However, we find that the welfare gap (equilibrium welfare minus optimal welfare) is increasing with turbulence.

The unemployment gap between the equilibrium allocation and the efficient one reaches 2.7 points when $\pi = 1/6$. It is worth noticing that equilibrium and efficient levels of unemployment move in opposite directions as turbulence increases. As stressed before, equilibrium unemployment increases with turbulence, because the share of trained workers, and job-offer arrival rates beyond $\tilde{a}$, decrease. At the optimum, all workers are trained and the vacancy rate is found to increase with π. The latter result means that the efficient job finding rate increases with turbulence, because the social planner internalizes the social cost of having – everything else being equal – a higher share of workers with obsolete knowledge, which implies a higher probability of paying the training costs (the ratio $u_1(a)/u(a)$ rises with π). In other words, when π rises, the overall costs paid for training are increasing; a way for the planner to limit this is to increase the job finding rate, and therefore to reduce unemployment.

Overall this therefore suggests that the normative issue related to turbulence is quantitatively important. We lastly investigate the design of optimal labor market policy in accordance with this illustration and then discuss the related implications as regards with existing policies.

4.3. On policy implications

Our quantitative illustration provides additional insights on the optimal policy implementation that we emphasized in Section 4.1: in addition to training subsidies that allow to promote training of low ability workers, another instrument is required to increase the number of vacancies. Externalities related to training investment lead to an inefficient ability threshold, but the choice by a firm to post or not a vacancy is made without internalizing the social cost of leaving type-2 workers in unemployment.

A subsidy rate on fixed training costs s allows to reach the efficient ability threshold for training. To obtain the efficient level of vacancies, the social planner can introduce an instantaneous flow of employment subsidies at level $v(\alpha)$ for jobs matched with workers of ability a.[21] To restore equilibrium efficiency, employment subsidy varies according to a. Importantly, due to Hosios condition, equilibrium unemployment is efficient for type-0 workers. Then, employment subsidies have to be paid only for workers whose ability is high enough to allow for training (type 1 or 2).

The last line of Table 3 first shows how the overall optimal subsidy evolves according to the turbulence parameter π. More precisely, it reports the expected total subsidy, expressed as a percentage of training costs $\left(\frac{s^\star \gamma_F + \frac{\nu^\star(a^\star)}{\delta}}{\gamma_F} = s^\star + \frac{\nu^\star(a^\star)}{\delta \gamma_F}\right)$,[22] received by a firm matched with a worker at the efficient ability threshold (the binding lower bound $\underline{a} = 1$ in our illustration). As expected, this total subsidy rate is found to increase with turbulence. This indeed comforts the idea that, in a context where it is optimal for all workers with obsolete knowledge to be trained, an increase in subsidies is required to reduce unemployment spell and therefore counteracts the additional social costs of a higher rate of human capital depreciation.

Fig. 6 then shows how the overall optimal subsidy is related to the ability level a. We express the flow of subsidies as a percentage of per period wage costs for type-1 workers; the corresponding flow of subsidies is therefore given by $s^\star \gamma_F \delta + \nu^\star(a)$. Interestingly, it comes that this optimal subsidy rate is decreasing with ability and wages. This mainly reflects that the weight of inefficiencies related to fixed training costs is relatively higher for the low ability workers than for the high ability ones.

We may then wonder how public intervention is implemented and matters in real world. First notice that the highest contributors to continuous vocational training of the employed in OECD countries are the employers, whose average contribution to the cost of continuous vocational training (CVT) is approximately 75% (see Bassanini et al. (2005)). More precisely, total monetary expenditures in CVT by firms represent 1.4% of labor costs in France (a little less than 1% in EU27).

This therefore suggests that private returns for firms of human capital investment are high, or that existing incentives and regulation are effective.

In practice, public intervention that aims at fostering lifelong learning takes various forms.

Public measures indeed provide both incentives and supports for the provision of continuous vocational training. This includes direct training services, regulation, procedures to ensure certifications, and also financial subsidies conditional on the fact that employees be trained (as developed in the present paper).

Most OECD countries are combining the different types of measures. From a public spending perspective, only the provision of training services and financial subsidies or tax incentives is costly for the government. In 2005, training measures represent 0.3% of GDP in France (which is also the average in Europe), and less than 0.05% in the US.

Those public expenditures on training are in general concentrated on specific individuals: in France, 60% for the young, 20% for the unemployed and the remaining 20% for the employed (DARES, 2012). The present paper deals with continuous vocational training for the employed, and does not address the specificities of the youth labor market. Concerning training of the unemployed, a companion result of this paper (available upon request) is that it is more worthwhile to subsidize firms to provide training, conditionally on hiring and training the worker (as it is considered), than subsidizing training during unemployment. The point is indeed that, despite being trained, the unemployed may face turbulence shock before getting a job, which means that the

[21] Appendix A.3 contains a formal description of the equilibrium with these two policy instruments.

[22] Since δ is the job destruction rate, $v(\alpha)\delta$ defines the total expected employment subsidies over the duration of the job.

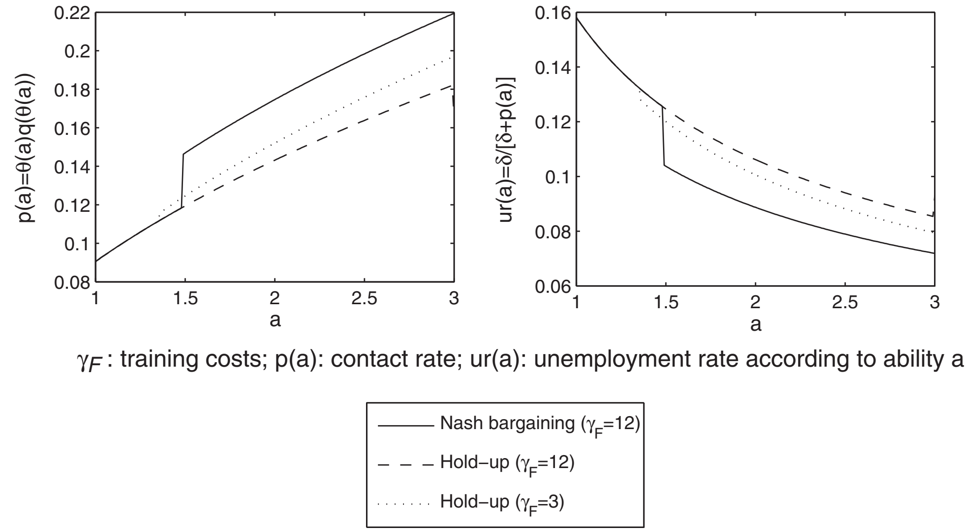

Fig. 4. Equilibrium: With *vs.* without hold-up. γ_F: training costs; $p(a)$: contact rate; $ur(a)$: unemployment rate according to ability a.

expected social return on the public investment in training is always lower than in the case where the subsidy is paid once the worker has been hired.

Our normative analysis does not deal with the diversity of policies and investments in continuous vocational training. It questions a political orientation that would systematically lead to increase training subsidies in periods of higher turbulence. This paper stresses that it is rather optimal to combine both training and employment subsidies, to help firms internalizing externalities related to their training decision. On the one hand, training subsidies allow some of the low-ability workers to benefit from training investments by employers. On the other hand, employment subsidies make the average unemployment duration shorter, and therefore reduce workers' exposure to human capital depreciation. The latter mechanism shows that high unemployment may have a huge social cost in times of increasing turbulence, so that reducing unemployment turns out to be – everything else being equal – of primer interest. To some extent, these results are in line with the general recommendation of the CEDEFOP[23] report of combining tax incentives to promote training with other policies in place.

5. Conclusion

Since the works of Ljungqvist and Sargent, it is now well established that an increase of the risk of human capital depreciation was an important driving force behind the rise of European unemployment, due to its interplay with generous unemployment benefits system. The main goal of this paper was to examine and discuss the normative incidence of turbulence on labor market outcomes by extending those previous works to account for endogenous training. Beyond the impact of human capital depreciation during unemployment spells on the well-known poaching externality, we emphasize the role of an unemployment externality. The optimal subsidy rate of training then depends on the relative size of those two externalities who exhibit opposite relationships with the turbulence parameter. Therefore, it is not necessarily clear cut whether to increase the subsidy rate of training in the context of higher turbulence. However, our quantitative illustration on the French economy emphasizes the primary role of reducing the unemployment duration, given the unemployment externality. This result is indeed found to be not only related to higher equilibrium unemployment but also lower efficient unemployment in turbulent times, which can account for an unemployment gap up to 2.7 points of percentage. This goal can be achieved through employment subsidies.

[23] CEDEFOP is a European Center which promotes the development of continuous vocational training.

Appendix AA.1. *Steady state equilibrium: Existence and uniqueness*

The steady state equilibrium consists in

- steady-state functions $p_0(\alpha)$ and $p(\alpha)$ that give the arrival rates of a job offer for each level of ability. We rewrite the free-entry Eqs. (6) and (7) using expression of unemployment levels (1), Bellman Eqs. (3)–(4) and wage Eqs. (17)–(20). Steady-state arrival rates $p_0(\alpha)$and $p(\alpha)$ are the respective solutions of the two following equations

$$\phi(p_0(a)) = a-b \tag{33}$$

$$\phi(p(a)) = (1+\Delta)(a-b)+\Delta b + \hat{\gamma}_F\left[\frac{\pi}{\pi+p}\psi_1(p(a))+\frac{p}{\pi+p}\psi_2(p(a))\right] \tag{34}$$

where

$$\phi(p) \equiv \frac{c(r+\delta+\beta p)}{(1-\beta)Q(p)}, \text{ with } Q(p) \text{ solution in } q \text{ of } p = M\left(\frac{p}{q},1\right). \tag{35}$$

$$\psi_1(p) \equiv -(r+\delta)+\delta\frac{\beta p}{r+\pi+\beta p} \tag{36}$$

$$\psi_2(p) \equiv \pi\frac{\beta p}{r+\pi+\beta p} \tag{37}$$

Table 2
Numerical results: The impact of hold-up.

	Nash	Hold-up			
	$\gamma_F = 12$	$\gamma_F = 12$	$\gamma_F = 3$	$\gamma_F = 3$	$\gamma_F = 3$
	$\pi = 1/6$	$\pi = 1/6$	$\pi = 1/6$	$\pi = 1/12$	$\pi = 1/24$
$F(\widetilde{a})$ in %	**50**	100	41	18	0
u in %	**11.5**	12.4	12.1	11.8	11.4

π: turbulence parameter; γ_F: training costs; $F(\widetilde{a})$: share of workers not trained; u: unemp. rate

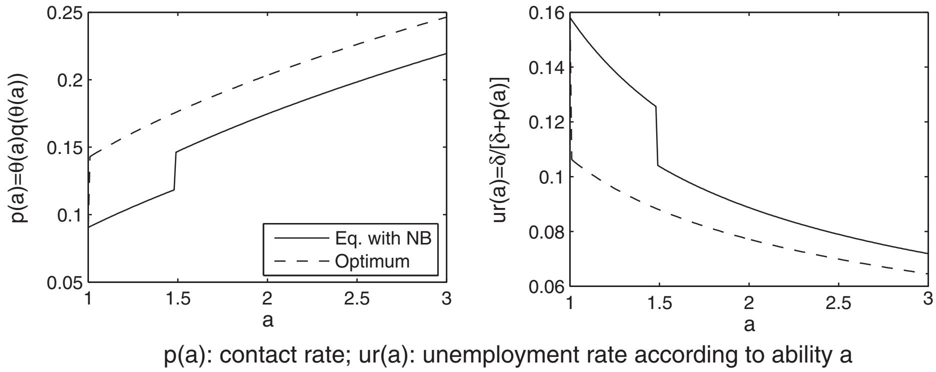

Fig. 5. Equilibrium without hold-up *vs.* optimum ($\gamma_F = 1/12$). $p(a)$: contact rate; $ur(a)$: unemployment rate according to ability a.

- an indicator function for any ability level a with value 0 if a-workers are never trained and with value 1 if a-workers are trained as soon as they have faced with human capital depreciation during the preceding unemployment spell. Then it takes value 1 if $J_1(a) - \hat{\gamma}_F = J_0(a)$, that is if

$$\left(1+\Delta-\frac{r+\delta+\beta p(a)}{r+\delta+\beta p_0(a)}\right)(a-b)+\Delta b+\psi_1(p(a))\hat{\gamma}_F\geq 0 \quad (38)$$

using Bellman Eqs. (3)–(4), and wage Eqs. (17)–(18).

As stated in Property 2, if the indicator function is equal to one for some ability level a, expected profit of the new entrant is higher than it would be if the a-workers were never trained. Consequently, everything else being equal, more firms enter the market and labor market tightness is lower.

We now turn to existence and uniqueness of the arrival rates.

Assumption 1. $\phi(0) < \underline{a} - b < \overline{a} - b < \phi(+\infty)$.

Noticing that $\phi(p)$ is strictly increasing leads to the following result.

Proposition 6. *Under Assumption 1, for any ability level a, there exists a unique steady state equilibrium arrival rate $p_0(\alpha)$, and $p_0'(a) > 0$.*

As stated in the next proposition, existence of the arrival rate $P(\alpha)$ can be easily derived from Assumption 1. But uniqueness is not always guaranteed. Indeed, the arrival rate has two opponent effects on the expected value of a filled job

$$\frac{u_1(a)}{u(a)}(J_1(a)-\hat{\gamma}_F)+\frac{u_2(a)}{u(a)}J_2(a)$$

or equivalently on the expected wage the firm will have to pay. Looking at the wage Eqs. (18) and (19), on the one hand, a higher arrival rate increases the worker's bargaining strength (through $x(\alpha)$), and this leads to higher expected wage. On the other hand, it also increases the gap between the values of unemployment $\mathcal{U}_2(a) - \mathcal{U}_1(a)$, and so lowers reservation wages of type-1 and type-2 workers. In order to state uniqueness, one may retain the following additional assumption:

Assumption 2. *For any ability level $a \in [\underline{a}, \overline{a}]$,*

- *the function*

$$h(p) \equiv \phi(p) - \hat{\gamma}_F\left[\frac{\pi}{\pi+p}\psi_1(p)+\frac{p}{\pi+p}\psi_2(p)\right]$$

is strictly convex with respect to p, for any $p > p_0(a)$.
- *the arrival rate $p_0(a)$ satisfies*

$$\phi(p_0(a)) - \hat{\gamma}_F\left[\frac{\pi}{\pi+p_0(a)}\psi_1(p_0(a))+\frac{p_0(a)}{\pi+p_0(a)}\psi_2(p_0(a))\right] < a-b.$$

Proposition 7. Under Assumptions 1 and 2, there exists a unique equilibrium arrival rates $p(a)$ strictly larger than $p_0(a)$, and such that $p'(a) > 0$.

Proof. We have to show that the equation $h(p) = (1+\Delta)(a-b)+\Delta b$ has a unique solution in p in the interval $[p_0(a), +\infty)$. From Assumptions 1 and 2, the function h goes from a value lower than $(1+\Delta)(a-b)+\Delta b$ to $+\infty$, and is strictly convex. The result follows. □

To keep the analysis simple, we consider cases with a threshold ability level $\tilde{a}$ below which workers are of type 0 and above which they are of type 1 or 2. A quick look at inequality (38) shows that the left-hand side is not necessarily increasing with respect to ability a. Since the productivity gain $\Delta\alpha$ is linear in a and the training cost is fixed, higher ability implies higher incentive for training. Nevertheless, higher ability also implies that jobs arrive to workers at higher rate whatever they are of type 0, 1 or 2. This leads to higher wages $w_0(\alpha)$. As mentioned above, the resulting effect on $w_1(\alpha)$ is not clear cut because of the two opponent effects: higher worker's bargaining strength and lower reservation

Table 3
Numerical results: Equilibrium *vs.* optimum.

	$\pi = 1/6$	$\pi = 1/12$	$\pi = 1/24$	$\pi \to 0$
Equilibrium with Nash bargaining				
$F(\tilde{a})$ in %	**50**	40.5	3	0
u in %	**11.5**	11.2	10.3	10.3
Welfare	**1.53**	1.57	1.66	1.69
Optimum				
$F(\alpha^*)$ in %	**0**	0	0	0
u^* in %	**8.8**	8.9	9.4	10.3
Welfare	1.61	1.64	1.67	1.69
Total subsidy rate $s^* + \frac{\nu^*(\alpha^*)}{\delta\gamma_F}$	0.83	0.58	0.06	0

π: turbulence parameter; F(.): share of workers not trained; u: unemp. rate.

Fig. 6. Turbulence and the optimal subsidy rate.

wage. Typically, we assume that Δ is large enough to ensure that the increase in productivity is more important than the effects of the variations in both arrival rates.

A.2. Social planner program

For an infinite lived economy, the social planner maximizes the present value of output net of search costs and training costs, subject to the same search frictions as faced by the decentralized economy. Since what happen to some a-ability level does not interact with other levels, we consider the social planner's problem for some ability a. We also leave the possibility for the social planner to choose the mass of new hired workers that will be trained. Let $\phi(a)$ denote the fraction of new hired workers that will be trained. We then have the following dynamic constraints which apply for each a (we drop the a-index for expositional convenience)

$$\dot{e}_0 = p_0(1-\phi)u_1 - \delta e_0$$
$$\dot{e} = p(\phi u_1 + u_2) - \delta e$$
$$\dot{u}_1 = \delta e_0 + \pi u_2 - [p\phi + p_0(1-\phi)]u_1$$
$$\dot{u}_2 = \delta e - (p+\pi)u_2.$$

We will show that the steady state optimum satisfies one of the two following situations:

- $\phi = 0$: new hired workers are never trained. Then all workers are of type-0 and $e = u_2 = 0$.
- $\phi = 1$: new hired workers are always trained if they have experienced human capital depreciation in the preceding unemployment spell. Then, workers are of type 1 or 2, and $e_0 = 0$.

The social objective for ability a writes

$$\int_0^{+\infty} e^{-rt}(fb + e_0(a-b) + e[(1+\Delta)a-b] - c[\theta_0(1-\phi)u_1 + \theta(\phi u_1 + u_2)] - \gamma_F p\phi u_1)dt$$

where we used the following expressions for the masses of type-0 vacancies $v_0 = \theta_0(1-\phi)u_1$ and other vacancies $v = \theta(\phi u_1 + u_2)$. The social planner's problem is to choose time paths of the control variables ϕ, θ, θ_0 and the state variables: e, u_1, u_2. Notice that e_0 is redundant since $e_0 = f - (e + u_1 + u_2)$.

To solve the social planner's problem, write down the current-valued Hamiltonian

$$H = fb + [f-(e+u_1+u_2)](a-b) + e[(1+\Delta)a-b] - c[\theta_0(1-\phi)u_1 + \theta(\phi u_1 + u_2)]$$
$$-\gamma_F p\phi u_1 + \lambda[p(\phi u_1 + u_2) - \delta e] + \mu_1[\delta(f-e-u_1-u_2) + \pi u_2 - [p\phi + p_0(1-\phi)]u_1]$$
$$+\mu_2[\delta e - (p+\pi)u_2]$$

where λ, μ_1 and μ_2 are the respective adjoint variables of e, u_1 and u_2.

A.2.1. Exogenous arrival rates and $c = 0$

Let us first consider the case with exogenous arrival rates as examined in Section 3.2 ($p_0 \le p$), and $c = 0$. The derivative of the Hamiltonian with respect to ϕ writes

$$\frac{\partial H}{\partial \phi} = [(\lambda - \gamma_F)p - (p-p_0)\mu_1]u_1.$$

Then, assuming $u_1 > 0$ at the optimum

- If $(\lambda - \gamma_F)p - (p-p_0)\mu_1 > 0$ (or equivalently $\lambda > \gamma_F + \left(1 - \frac{p_0}{p}\right)\mu_1$), $\frac{\partial H}{\partial \phi} \ge 0$ and then $\phi = 1$.
- If $(\lambda - \gamma_F)p - (p-p_0)\mu_1 < 0$, $\frac{\partial H}{\partial \phi} < 0$ then $\phi = 1$.

Let us now write the optimality conditions with respect to state variables

$$\frac{\partial H}{\partial e} = -\dot{\lambda} + r\lambda \Leftrightarrow \Delta a - (\lambda + \mu_1 - \mu_2)\delta = -\dot{\lambda} + r\lambda$$
$$\frac{\partial H}{\partial u_1} = -\dot{\mu}_1 + r\mu_1 \Leftrightarrow -(a-b) + (\lambda - \gamma_F)p\phi - \mu_1[\delta + p\phi + p_0(1-\phi)] = -\dot{\mu}_1 + r\mu_1$$
$$\frac{\partial H}{\partial u_2} = -\dot{\mu}_2 + r\mu_2 \Leftrightarrow -(a-b) + \lambda p + \mu_1(\pi - \delta) - (p+\pi)\mu_2 = -\dot{\mu}_2 + r\mu_2.$$

In steady state ($\dot{\lambda} = \dot{\mu}_1 = \dot{\mu}_2 = 0$), $\frac{\partial H}{\partial \phi}$ has the same sign as

$$\Delta a + \left(-(r+\delta) + \frac{\delta p}{r+\pi+p}\right)\gamma_F + (a-b)\frac{r+\delta}{r+\delta+p_0}\frac{p-p_0}{p}. \quad (39)$$

Hence, considering $\frac{\partial H}{\partial \phi} = 0$ gives the threshold ability value at the optimum, as defined by α^* in Eq. (28).

A.2.2. Endogenous arrival rates and $c > 0$

Optimality conditions with respect to the state variables write

$$\frac{\partial H}{\partial e} = -\dot{\lambda} + r\lambda \Leftrightarrow \Delta a - (\lambda + \mu_1 - \mu_2)\delta = -\dot{\lambda} + r\lambda \quad (40)$$

$$\frac{\partial H}{\partial u_1} = -\dot{\mu}_1 + r\mu_1 \Leftrightarrow -(a-b) - c[\theta_0(1-\phi) + \theta\phi] + (\lambda - \gamma_F)p\phi - \mu_1[\delta + p\phi + p_0(1-\phi)] = -\dot{\mu}_1 + r\mu_1 \quad (41)$$

$$\frac{\partial H}{\partial u_2} = -\dot{\mu}_2 + r\mu_2 \Leftrightarrow -(a-b) - c\theta + \lambda p + \mu_1(\pi - \delta) - (p+\pi)\mu_2 = -\dot{\mu}_2 + r\mu_2. \quad (42)$$

With endogenous arrival rates, we have to consider the derivatives of the Hamiltonian with respect to labor market tightness θ and θ_0. Then, we get the two following cases

- If $\frac{\partial H}{\partial \phi} \ge 0$, then $\phi = 1$ and labor market tightness θ must satisfy

  $$\frac{\partial H}{\partial \theta} = -c(u_1 + u_2) - \gamma_F p'(\theta)u_1 + \lambda[p(u_1+u_2) - \delta e] - \mu_1 p'(\theta)u_1 - \mu_2 p'(\theta)u_2 = 0$$

  where the adjoint variables λ, μ_1 and μ_2 can be computed from Eqs. (40), (41) and (42).
- If $\frac{\partial H}{\partial \phi} < 0$, then $\phi = 0$ and θ_0 must satisfy

  $$\frac{\partial H}{\partial \theta_0} = -\left[c + \mu_1 p_0'(\theta_0)\right]u_1 = 0$$

  where the adjoint variable μ_1 can also be computed from the optimality conditions with respect to state variables.

In order to determine whether a-ability workers are of type 0 or not, we analyze the sign of the derivative of the current-valued Hamiltonian with respect to

$$\frac{\partial H}{\partial \phi} = [-c(\theta - \theta_0) + p(\lambda - \gamma_F) - (p-p_0)\mu_1]u_1.$$

In steady state, Eqs. (40), (41) and (42) imply that $\frac{\partial H}{\partial \phi}$ has the same sign as

$$\Delta a + \left(-(r+\delta) + \frac{\delta p}{r+\pi+p}\right)\gamma_F + \frac{(r+\delta)(p-p_0)}{(r+\delta+p_0)p}(a-b)$$
$$+ (r+\delta)c\frac{\theta}{p}\left(\frac{\theta_0}{r+\delta+p_0}acr + \delta + p\theta - 1\right).$$

Considering again $\frac{\partial H}{\partial \phi} = 0$ gives the condition that determines the optimal threshold ability value a^* in this general case (Eq. (32) where it is assumed $r = 0$). Computing μ_1 and μ_2 then allows to state Eqs. (30) and (31).

A.3. The labor market equilibrium with employment subsidies

Intertemporal values of jobs filled with type-1 and type-2 workers satisfy

$$rJ_i(a) = (1+\Delta)a - w_i(a) + \nu(a) - \delta(J_i(a) - V(a)), i = \{1,2\}$$

where $v(a)$ is the employment subsidy that depends on ability a. Type-0 wage is obviously unchanged whereas, assuming $r = 0$, wage equation for type-1 becomes

$$w_1(a) = \beta\left(\frac{\delta + p(a)}{\delta + \beta p(a)}\right)[(1+\Delta)a + \nu(a) - \delta\gamma_F(1-s)] + \left(\frac{(1-\beta)\delta}{\delta + \beta p(a)}\right)\left[b - \gamma_F(1-s)\left(\frac{\beta p(a)}{\pi + \beta p(a)}\right)\right].$$

Accordingly, the labor market equilibrium with policy instruments satisfies the following set of equations:

$$\frac{c}{q_0(a)} = \frac{(1-\psi)(a-b)}{\delta + \psi p_0(a)}, \quad \text{for } a<\tilde{a}$$

$$\frac{c}{q(a)} = \frac{(1-\psi)((1+\Delta)a + \nu(a) - b)}{\delta + \psi p(a)} - \frac{(1-\psi)\gamma_F(1-s)}{\delta + \psi p(a)}\left(\frac{\pi}{\pi + p(a)}\right)\left[\delta - \frac{(\delta + p(a))\psi p(a)}{\pi + \psi p(a)}\right], \quad \text{for } a \geq \tilde{a}$$

$$\Delta\tilde{a} = \gamma_F(1-s)\left(\frac{\pi\delta}{\pi + \psi p(\tilde{a})}\right) + (\tilde{a} - b)\psi\left(\frac{p(\tilde{a}) - p_0(\tilde{a})}{\delta + \psi p_0(\tilde{a})}\right) - \nu(a).$$

Taking $\tilde{a} = a^\star$ and $\theta(a) = \theta^*(a)\ \forall a$, one can compute the optimal values of the subsidies s^* and $v^*(a)$, for any $a \geq a^*$. For $a < a^*$, the Hosios condition guarantees that the equilibrium value of $\theta_0(a)$ is efficient.

References

Acemoglu, D., 1997. Training and innovation in an imperfect labor market. Rev. Econ. Stud. 64, 445–464.

Acemoglu, D., Pischke, J.-S., 1998. Why do firms train? Theory and evidence. Q. J. Econ. 113, 79–119.

Acemoglu, D., Pischke, J.-S., 1999a. The structure of wages and investment in general training. J. Polit. Econ. 107, 539–572.

Acemoglu, D., Pischke, J.-S., 1999b. Beyond Becker: Training in imperfect labor markets. Econ. J. 109, 112–142.

Acemoglu, D., Shimer, R., 1999. Holdups and efficiency with search frictions. Int. Econ. Rev. 40 (4), 827–849.

Bassanini, A., Booth, A., Brunello, G., De Paloa, M., Leuven, E., 2005. Workplace training in Europe. IZA Discussion Papers 1640.

Brunello, G., Comi, S.L., Sonedda, D., 2012. Training subsidies and the wage returns to continuing vocational training: Evidence from Italian regions. Labour Econ. 19 (3), 361–372.

Chéron, A., 2005. Efficient vs. equilibrium unemployment with match-specific costs. Econ. Lett. 88 (2), 176–183.

Cunha, F., Heckman, J., Lochner, L., 2006. Interpreting the evidence on life cycle skill formation. Handbook of the Economics of EducationElsevier.

DARES, 2012. La dépense nationale pour la formation professionnelle continue et l'apprentissage en 2010. Document d'études, pp. 2012–2168.

Decreuse, D., Granier, P., 2013. Unemployment benefits, job protection, and the nature of educational investment. Labour Econ. 23, 20–29.

Den Haan, W., Haefke, C., Ramey, V., 2005. Turbulence and unemployment in a job matching model. J. Eur. Econ. Assoc. 3 (6), 1360–1385.

Farber, H., 2005. What Do We Know about Job Loss in the United States? Evidence from the Displaced Worker Survey, 1984–2004. Princeton University Working Paper., 498.

Hairault, J.-O., Le Barbanchon, T., Sopraseuth, T., 2012. The cyclicality of the separation of job finding rates in France. IZA Discussion Papers, p. 6906.

Jacobson, L.S., LaLonde, R.J., Sullivan, D.G., 1993. Earnings losses of displaced workers. Am. Econ. Rev. 83, 685–709.

Leuven, E., 2005. The economics of private sector training: a survey of the literature. J. Econ. Surv. 19, 91–111.

Ljungqvist, L., Sargent, T.J., 1998. The European unemployment dilemma. J. Polit. Econ. 106, 514–550.

Ljungqvist, L., Sargent, T.J., 2004. European unemployment and turbulence revisited in a matching model. J. Eur. Econ. Assoc. 2, 456–468.

Ljungqvist, L., Sargent, T.J., 2007. Understanding European unemployment with matching and search-island models. J. Monet. Econ. 54 (8), 2139–2179.

Malcomson, J.M., 1997. Contracts, hold-up, and labor markets. J. Econ. Lit. 35 (4), 1916–1957.

Ok, W., Tergeist, P., 2003. Improving workers' skill: analytical evidence and the role of the social partners. *OECD Social* Employment and Migration Working Papers, p. 10.

Picchio, M., van Ours, J.C., 2011. Market imperfections and firm-sponsored training. Labour Econ. 18 (5), 712–722.

Pissarides, C.A., 1992. Loss of skill during unemployment and the persistence of employment shocks. Q. J. Econ. 107 (4), 1371–1391.

Stevens, M., 1994a. A theoretical model of on-the-job training with imperfect competition. Oxf. Econ. Pap. 46 (4), 537–562.

Stevens, M., 1994b. An investment model for the supply of training by employers. Econ. J. 104, 556–570.

Tripier, F., 2011. The efficiency of training and hiring with intrafirm bargaining. Labour Econ. 18 (4), 527–538.

Wasmer, E., 2006. Interpreting Europe–US labor market differences: the specificity of human capital investments. Am. Econ. Rev. 96 (3), 811–831.